Plenary Lectures

Loewner Matrices

Rajendra Bhatia, Indian Statistical Institute
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Mon 9:00, Auditorium

Let f be a smooth function on $\mathbb{R}$. The divided difference matrices whose (i, j) entries are

$$\left[\frac{f(\lambda_i) - f(\lambda_j)}{\lambda_i - \lambda_j}\right]$$

$\lambda_1, \ldots, \lambda_n \in \mathbb{R}$ are called *Loewner matrices.* In a seminal paper published in 1934 Loewner used properties of these matrices to characterise operator monotone functions. In the same paper he established connections between this matrix problem, complex analytic functions, and harmonic analysis. These elegant connections sent Loewner matrices into the background. Some recent work has brought them back into focus. In particular, characterisation of operator convex functions in terms of Loewner matrices has been obtained. In this talk we describe some of this work. The talk will also serve as an introduction to some more recent and more advanced topics being presented by some other speakers in this conference.

Matrices and Indeterminates

Richard A. Brualdi , University of Wisconsin - Madison, USA
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Thu 9:00, Auditorium

An expository talk will be given on matrices some of whose entries are indeterminates over a field. The talk will include some recent joint work on such matrices with Zejun Huang and Xingzhi Zhan, and some recent joint work on combinatorial batch codes with K.P. Kiernan, S.A. Meyer, and M.W. Schroeder, some of which can be placed in the context of such matrices.

Potential Stability and Related Spectral Properties of Sign Patterns

Pauline van den Driessche, University of Victoria, B.C. Canada
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Mon 14:00, Auditorium

An $n \times n$ sign pattern $\mathcal{S} = [s_{ij}]$ has $s_{ij} \in \{+, -, 0\}$ and gives rise to an associated sign pattern class of matrices

$$Q(\mathcal{S}) = \{A = [a_{ij}] : a_{ij} \in \mathbb{R},\ \text{sign}\ a_{ij} = s_{ij}\ \forall i, j\}.$$

Sign pattern $\mathcal{S}$ has inertia (n_+, n_-, n_0) with $n_+ + n_- + n_0 = n$ if there exists a matrix $A \in Q(\mathcal{S})$ with this inertia. In particular $\mathcal{S}$ is potentially stable if it allows inertia $(0, n, 0)$, i.e., there exists a matrix $A \in Q(\mathcal{S})$ with each eigenvalue having a negative real part. Since its introduction in the context of qualitative economics over 40 years ago, the problem of characterizing potential stability of sign patterns remains unsolved except for special classes of sign patterns, e.g., when the digraph associated with the pattern can be represented by a tree. Elaborating on [1], known necessary or sufficient conditions for potential stability are reviewed, techniques for constructing potentially stable patterns described and open problems stated. Some results are also given for sign patterns that allow more general inertias, and those that allow any spectrum of a real matrix.

[1] M. Catral, D.D. Olesky and P. van den Driessche, Allow problems concerning spectral properties of sign patterns, Linear Algebra and its Applications, 430, pp. 3080-3094, 2009.

Computing the Action of the Matrix Exponential, with an Application to Exponential Integrators

Nicholas J. Higham , University of Manchester, UK
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Tue 14:00, Auditorium

A new algorithm is developed for computing $e^{tA}B$, where A is an $n \times n$ matrix and B is $n \times n_0$ with $n_0 \ll n$. The algorithm works for any A, its computational cost is dominated by the formation of products of A with $n \times n_0$ matrices, and the only input parameter is a backward error tolerance. The algorithm can return a single matrix $e^{tA}B$ or a sequence $e^{t_k A}B$ on an equally spaced grid of points t_k. It uses the scaling part of the scaling and squaring method together with a truncated Taylor series approximation to the exponential. It determines the amount of scaling and the Taylor degree using the recent analysis of Al-Mohy and Higham [*SIAM J. Matrix Anal. Appl.* 31 (2009), pp. 970-989], which provides sharp truncation error bounds expressed in terms of the quantities $\|A^k\|^{1/k}$ for a few values of k, where the norms are estimated using a matrix norm estimator. Shifting and balancing are used as preprocessing steps to reduce the cost of the algorithm. Numerical experiments show that the algorithm performs in a numerically stable fashion across a wide range of problems, and analysis of rounding errors and of the conditioning of the problem provides theoretical support. Experimental comparisons with two Krylov-based MATLAB codes show the new algorithm to be sometimes much superior in terms of computational cost and accuracy. An important application of the algorithm is to exponential integrators for ordinary differential equations. It is shown that the sums of the form $\sum_{k=0}^{p} \varphi_k(A)u_k$ that arise in exponential integrators, where the φ_k are related to the exponential function, can be expressed in terms of a single exponential of a matrix of dimension $n + p$ built by augmenting A with additional rows and columns, and the algorithm of this paper can therefore be employed.

Joint work with Awad H. Al-Mohy (University of Manchester)

Nonsymmetric Algebraic Riccati Equations Associated with M-matrices: Theoretical Results and Algorithms

B. Meini, University of Pisa, Italy
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Fri 14:00, Auditorium

Nonsymmetric algebraic Riccati equations (NARE) are nonlinear matrix equations of the kind

$$C + XA + DX - XBX = 0,$$

where the unknown X is an $m \times n$ matrix and A, B, C, D are matrices of appropriate size. We focus the attention on NAREs whose block coefficients are such that the matrix

$$M = \begin{bmatrix} A & -B \\ C & D \end{bmatrix}$$

is either a nonsingular M-matrix, or a singular irreducible M-matrix. This class of equations arises in a large number of applications, ranging from fluid queues models to transport theory. The solution of interest is the minimal nonnegative one, i.e., the nonnegative solution $X_{\min}$ such that $X_{\min} \leq X$ for any other nonnegative solution X, where the ordering is component-wise.

In this talk we present theoretical properties of the NARE and numerical methods for the computation of the minimal nonnegative solution $X_{\min}$. Particular emphasis is given to the properties of the invariant subspaces, and to the techniques used to trasform the eigenvalues of a pencil, keeping unchanged the invariant subspaces. Concerning numerical methods, special attention is addressed to structure-preserving iterative algorithms; connections between the cyclic reduction algorithm and the structure-preserving doubling algorithm (SDA) are pointed out.

Potpourri of Quasiseparable Matrices

Vadim Olshevsky (LAMA Speaker), University of Connecticut, USA
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Fri 9:00, Auditorium

In this talk we provide a survey of recent results on quasiseparable matrices in three different areas. We start with CMV matrices that garner a lot of attention in the orthogonal polynomials community. Our quasiseparable approach allows one to generalize some already clasical results to a wider class of matrices. The second topic is application of quasiseparable matrices to new digital filter structures. Again, quasiseparable approach allows one to generalize the celebrated Markel-Grey filter structure and Kimura structure. Finally, we desribe the results of error analysis of several published quasiseparable system solvers that indicate that only one of them is a provably backward stable algorithm while the others are not.

Joint work with Tom Bella, Forilan Dopico, Gil Strang and Pavel Zhlobich

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Moments, model reduction and nonlinearity in solving linear algebraic problems

Z. Strakoš, Charles University, Prague, Czech Republic
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Thu 14:00, Auditorium

Krylov subspace methods play an important role in many areas of scientific computing, including numerical solution of linear algebraic systems arising from discretisation of partial differential or integral equations. By their nature they represent *model reductions based on matching moments*. Such view naturally complements, in our opinion, the standard description using the projection processes framework, and it shows their highly nonlinear character.

We present three examples that link algebraic views of problems with views from related areas of mathematics:

- Matching moments reduced order modeling in approximation of large-scale linear dynamical systems is linked with the classical work on moments and continued fractions by Chebyshev and Stieltjes, and with development of the conjugate gradient method by Hestenes and Stiefel.
- We show that Gauss-Christoffel quadrature for a small number of quadrature nodes can be highly sensitive to small changes in the distribution function, and we relate the sensitivity of Gauss-Christoffel quadrature to the convergence properties of the CG and Lanczos methods in exact and in finite precision arithmetic.
- Based on the method of moments, we show how the information from the Golub-Kahan iterative bidiagonalization can be used for estimating the noise level in discrete ill-posed problems.

Joint work with I. Hnětynková (Charles University, Prague), D. P. O'Leary (University of Maryland), M. Plešinger (Technical University, Liberec), P. Tichý (Academy of Sciences, Prague)

Modifications to block Jacobi with overlap to accelerate convergence of iterative methods for banded matrices.

Daniel B. Szyld , Temple University, Philadelphia, USA
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Tue 9:00, Auditorium

Classical Schwarz methods and preconditioners subdivide the domain of a partial differential equation into subdomains and use Dirichlet or Neumann transmission conditions at the artificial interfaces. Optimizable Schwarz methods use Robin (or higher order) transmission conditions instead, and the Robin parameter can be optimized so that the resulting iterative method has an optimal convergence rate. The usual technique used to find the optimal parameter is Fourier analysis; but this is only applicable to certain domains, for example, a rectangle.

In this talk, we present a completely algebraic view of Optimizable Schwarz methods, including an algebraic approach to find the optimal operator or a sparse approximation thereof. This approach allows us to apply this method to any banded or block banded linear system of equations, and in particular to discretizations of partial differential equations in two and three dimensions on irregular domains. This algebraic Optimizable Schwarz method is in fact a version of block Jacobi with overlap, where certain entries in the matrix are modified.

With the computable optimal modifications, we prove that the Optimizable Schwarz method converges in two iterations for the case of two subdomains. Similarly, we prove that when we use an Optimizable Schwarz preconditioner with this optimal modification, the underlying Krylov subspace method (e.g., GMRES) converges in two iterations. Very fast convergence is attained even when the optimal operator is approximated by a sparse transmission matrix. Numerical examples illustrating these results are presented.

Joint work with Martin Gander and Sébastien Loisel (University of Geneva)

Linear algebraic foundations of the operational calculi

Luis Verde-Star , Universidad Autónoma Metropolitana, Mexico City, Mexico
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Wed 9:00, Auditorium

One of the most common problems in Applied Mathematics consists in finding solutions f of a linear functional equation of the form $w(L)f = g$, where w is a polynomial, L is an operator, and g is a known function. Differential and difference equations with constant or variable coefficients are included among such equations.

We will construct a vector space $\mathcal{F}$ of formal Laurent series generated by a set $\{p_k : k \in \mathbb{Z}\}$, with the natural multiplication induced by $p_k p_n = p_{k+n}$, and an operator L defined by $Lp_0 = 0$ and $Lp_k = p_{k-1}$ for $k \neq 0$. Then we will show how to solve the general equation $w(L)f = g$ in the space $\mathcal{F}$ using only elementary linear algebraic ideas.

It turns out that $\mathcal{F}$ is a sort of universal model for the solution of many types of linear functional equations. Giving a suitable particular meaning to the generators p_k we obtain a concrete space where L becomes a given differential-like operator, and the multiplication in $\mathcal{F}$ becomes a "convolution" in the concrete space. For example, if $p_k = t^k/k!$ then L becomes differentiation with respect to t. We can also obtain operators of the form $u(t)D + v(t)I$, and difference operators.

Our development clarifies how the basic concepts of the operational calculi appear in a natural way. For example, we explain the connection between convolutions and divided differences, the role played by quasi-polynomials, and how the transform methods become unnecessary in most cases. Our results generalize some of the ideas presented in [1].

[1] L. Verde-Star, An algebraic approach to convolutions and transform methods, Adv. in Appl. Math. **19**, 117–143, 1997.

Special Lectures

Krylov Subspace Approximations of the Action of Matrix Functions for Large-Scale Problems

Oliver Ernst, TU Bergakademie Freiberg, Germany
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Tue 9:45, Auditorium

We present an overview of recent progress in the development of Krylov subspace methods for the approximation of expressions of the form $f(A)b$, where A is a large, sparse or structured matrix, f is a function such that $f(A)$ is defined and b a given vector. Such an action of a matrix function on a vector is a fundamental computational task in many applications, of which the most prominent is the matrix exponential occurring in initial value problems for systems of ordinary differential equations or semidiscretized partial differential equations. Special emphasis will be given to restarting techniques [1], rational Krylov subspace approximations [2], error estimates and convergence theory. The performance of these techniques will be illustrated for a large-scale problem arising in geophysical exploration [3].

[1] M. Eiermann and O. G. Ernst, A restarted Krylov subspace method for the evaluation of matrix functions. SIAM J. Numer. Anal. 44:2481–2504 (2006)
[2] S. Güttel. Rational Krylov Methods for Operator Functions, doctoral thesis, TU Bergakademie Freiberg (2010).
[3] R. Börner, O. G. Ernst and K. Spitzer. Fast 3D simulation of transient elec- tromagnetic fields by model reduction in the frequency domain using Krylov subspace projection. Geophys. J. Int., 173:766–780 (2008).

Joint work with M. Afanasjew, S. Güttel, M. Eiermann (TU Bergakademie Freiberg)

Zeros of entire functions: from René Descartes to Mark Krein and beyond

Olga Holtz, University of California, Berkeley, USA
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Fri 9:45, Auditorium

The central question of many classical investigations, going back to Descartes, Newton, Euler, and others, is finding zeros of entire and meromorphic functions, given some standard representation of such a function, e.g., its coefficients in some standard basis. Special questions of this type include zero localization with respect to a given curve (e.g., stability and hyperbolicity), behavior of zeros under special maps (e.g., differentiation, Hadamard product), and relations among roots of function families (e.g., orthogonal polynomials). The point of this talk is to give an overview of matrix and operator methods in this area, emphasizing beautiful old and new connections between algebra and analysis. The novel results in this talk are joint with Mikhail Tyaglov.

Multilinear Algebra and its Applications

Lek-Heng Lim, University of California, Berkeley, USA, and University of Chicago
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Mon 9:45, Auditorium

In mathematics, the study of multilinear algebra is largely limited to properties of a whole space of tensors — tensor products of k vector spaces, modules, vector bundles, Hilbert spaces, operator algebras, etc. There is also a tendency to take an abstract coordinate-free approach. In most applications, instead of a whole space of tensors, we are often given just a single tensor from that space; and it usually takes the form of a hypermatrix, i.e. a k-dimensional array of numerical values that represents the tensor with respect to some coordinates/bases determined by the units and nature of measurements. How could one analyze this one single tensor then?

If the order of the tensor $k = 2$, then the hypermatrix is just a matrix and we have access to a rich collection of tools: rank, determinant, norms, singular values, eigenvalues, condition number, pseudospectrum, RIP constants, etc. This talk is about the case when $k > 2$.

We will see that one may often define higher-order analogues of common matrix notions rather naturally: tensor ranks, hyperdeterminants, tensor norms (Hilbert-Schmidt, spectral, Schatten, Ky Fan, etc), tensor eigenvalues and singular values, etc. We will discuss the utility as well as difficulties of various tensorial analogues of matrix problems. In particular we shall look at how tensors arise in a variety of applications including: computational complexity, control engineering, holographic algorithms, mathematical biology, neuroimaging, numerical analysis, quantum computing, signal processing, spectroscopy, and statistics. Time permitting, we will also describe a few exciting recent breakthoughs, most notably Landsberg's settlement of the border rank of 2×2 matrix multiplications and Friedland's resolution of the Salmon conjecture.

Evolution of MATLAB

Cleve Moler, The MathWorks
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Thu 9:45, Auditorium

We show how MATLAB has evolved over the last 25 years from a simple matrix calculator to a powerful technical computing environment. We demonstrate several examples of MATLAB applications. We conclude with a few comments about future developments, including Parallel MATLAB.

Cleve Moler is the original author of MATLAB and one of the founders of the MathWorks. He is currently chairman and chief scientist of the company, as well as a member of the National Academy of Engineering and former president of the Society for Industrial and Applied Mathematics.
See `http://www.mathworks.com/company/aboutus/founders/clevemoler.html`.

Linear ALgebra Meets Lie Algebra

Beresford N. Parlett, University of California, Berkeley, USA
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Wed 9:45, Auditorium

We examine the matrix congruence class $A \to GA\text{inv}(G)$ in which are preserved not only the eigenvalues of A but the eigenvalues of all the leading principal submatrices of A. We show connections both to the Kostant-Wallach theory in Lie Algebra and to the recent work of Olshevsky, Zhlobich, and Strang on Green's matrices. This is joint work with Noam Shomron.

Invited Minisymposia

Structured Matrices

Yuli Eidelman, Tel Aviv University, Israel
Lothar Reichel, Kent State University, USA
Marc Van Barel, Katholieke Universiteit Leuven

Fifteen years of structured matrices

F. Di Benedetto, Dipartimento di Matematica, Università di Genova
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Tue 11:00, Room Pacinotti

The expression *structured matrices* appeared for the first time in a conference title in 1995, specifically in the session "Algorithms for Structured Matrices", organized within the SPIE conference, held in San Diego (USA) [1, Session 6] and in the "Minisymposium on Structured Matrices" within the ILAS conference, held in Atlanta (USA) [2]. These first experiences led to the organization, in 1996, of the first two conferences specifically devoted to structured matrices: "International Workshop on Numerical Methods for Structured Matrices in Filtering and Control", held in Santa Barbara (USA) [3] and "Toeplitz Matrices: Structures, Algorithms and Applications" held in Cortona (Italy) [4].

The organization of specific conferences on structured matrices has given the opportunity to meet together researchers working on theoretical and computational properties of structured matrices, and researchers working on applications. This exchange of experts from different fields has led to strong benefits to researches interested in structured matrices.

This anniversary gives the opportunity to reflect on the state-of-art in the research involving matrix structures in the last 15 years. The aim of this talk is to survey some key results achieved along different directions, paying special attention to the significant contribution of the italian research group on structured numerical linear algebra, having its main site in Pisa. In particular, the four editions of the Cortona Workshop offer a privileged point of view in this context. Some pointers to future research perspectives will be also given.

[1] Advanced Signal Processing Algorithms, SPIE Vol. 2563, pp. 266–313, 1995.
[2] Proceeding of the Fifth Conference of the International Linear Algebra Society, Atlanta, Georgia (1995). Linear Algebra Appl. Vol. 254, pp. 1–5, 1997.
[3] http://www-control.eng.cam.ac.uk/extras/conferences/WNMSMFC96
[4] Calcolo Vol. 33, pp. 1–10, 1996.

Joint work with S. Serra Capizzano (Università dell'Insubria)

An enhanced plane search scheme for complex-valued tensor decompositions

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Tue 11:50, Room Pacinotti

A third-order tensor $T_1 \in \mathbb{C}^{l\times m\times n}$ is rank one if it can be written as the outer product of three nonzero vectors, i.e., $T_1 = a_1 \circ b_1 \circ c_1$. The CANDECOMP/PARAFAC (CP) decomposition writes a given tensor T as a sum of R rank-one tensors $T_i = a_i \circ b_i \circ c_i$. Factor matrices A, B and C are obtained by stacking the component vectors, e.g., $A = (a_1 a_2 \dots a_R)$.

The most popular methods for computing CP are of the alternating least squares type (ALS). These methods have many drawbacks: they can take many iterations to converge, they are not guaranteed to converge to a global minimum or even a stationary point, and the final solution can heavily depend on the starting value. Moreover, these algorithms ignore structure of the given tensor, such as symmetry.

Recently, an enhanced line search (ELS) procedure has been proposed for improving ALS. In ELS a system of polynomial equations in two variables needs to be solved in each iteration. This subproblem can be much more expensive than the initial ALS iteration.

We propose an enhanced plane search (EPS) as an alternative to ELS. The corresponding polynomial subproblem is much easier to solve. We combine EPS with the single-step least squares algorithm (SSLS) that has recently been proposed for the computation of a CP with factors A, B, C such that $A = B$ and A proportional to $\overline{C}$. The original SSLS algorithm is very cheap, but its convergence is not guaranteed. Our algorithm always converges and has better performance.

Joint work with L. De Lathauwer (K.U.Leuven: Campus Kortrijk and E.E. Dept. (ESAT), Belgium)

Structured perturbation theory of LDU factorization and accurate computations for diagonally dominant matrices

Froilán M. Dopico, Universidad Carlos III de Madrid, Leganés, Spain
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Tue 17:35, Room Pacinotti

If an LDU factorization with well-conditioned L and U factors of a given matrix A can be accurately computed, then the SVD of A can also be accurately computed [1] and the system of equations $Ax = b$ can be accurately solved for almost all right hand sides [2], independently of the magnitude of the traditional condition number of A. These facts have motivated in the last years the development of *structured* algorithms for computing accurate LDU factorizations of structured matrices with well-conditioned L and U factors. One of the most important classes of matrices arising in applications is the class of *diagonally dominant matrices*, and recently an *structured* algorithm for computing their LDU factorizations has been introduced in [3]. Unfortunately, the best error bound proven in [3] for this algorithm is $6\,n\,8^{(n-1)}\,\epsilon$, where n is the size of the matrix and ϵ is the unit roundoff. This bound is completely useless for sizes as small as $n = 20$ in double precision. We present in this talk a new structured perturbation theory for the LDU factorization of diagonally dominant matrices parameterized in a certain way, that allows us to prove an error bound $14\,n^3\,\epsilon$ for the LDU factorization computed with the algorithm in [3]. These results guarantee accurate computations of SVD and system solutions for any diagonally dominant matrix.

[1] J. Demmel, M. Gu, S. Eisenstat, I. Slapničar, K. Veselić, and Z. Drmač, *Computing the singular value decomposition with high relative accuracy*, Linear Algebra Appl. **299**(1–3), 21–80 (1999)
[2] F. M. Dopico and J. M. Molera, *Accurate solutions of structured linear systems*, in preparation.
[3] Q. Ye, *Computing singular values of diagonally dominant matrices to high relative accuracy*, Math. Comp. **77**(264), 2195–2230 (2008)

Joint work with Plamen Koev (San Jose State University, CA, USA)

Using quasiseparable structure for polynomial roots computations

Y. Eidelman, Tel Aviv University, Israel
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Wed 12:40, Room Pacinotti

The effective tool to compute all the roots of a polynomial is to determine the eigenvalues of the corresponding companion matrix using the QR iteration method. It turns out that the companion matrix belongs to a class of structured matrices which is invariant under QR iterations. Every matrix in this class has quasiseparable structure. This structure may be used to develop fast algorithms to compute eigenvalues of companion matrices. We discuss implicit fast QR eigenvalue algorithms solving this problem. The obtained algorithm is of complexity $O(N^2)$ in contrast to $O(N^2)$ for non-structured methods. The presentation is mainly based on the results of papers [1] and [2].

[1] S. Chandrasekaran, M. Gu, J. Xia, J. Zhu, A fast QR Algorithm for Companion Matrices, Operator Theory: Advances and Applications. 179, pp 111-143, 2007.
[2] D. A. Bini, P. Boito, Y. Eidelman, L. Gemignani, I. Gohberg, A Fast Implicit QR Algorithm for Companion Matrices, LAA. 432, 8, pp. 2006-2031, 2010.

Rank-structured matrix technology for solving nonlinear equations

L. Gemignani, University of Pisa, Italy
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Wed 11:25, Room Pacinotti

In this talk we discuss the use of rank-structured matrix methods for solving certain nonlinear equations arising in applications.

Inverses of pentadiagonal recursion matrices

Carl Jagels, Hanover College, Hanover, IN 47243, USA
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Tue 16:45, Room Pacinotti

Schemes for approximating matrix functions of the form $f(A)v$, where A is a a large, possibly sparse, symmetric matrix, based on projections onto the extended Krylov subspace are currently being explored. Short recursion relations for generating orthonormal bases of extended Krylov subspaces of the type $\mathbb{K}^{m,mi+1}(A) = \text{span}\{A^{-m+1}v, \ldots, A^{-1}v, v, Av, \ldots, A^{mi}v\}$, $m = 1, 2, 3, \ldots$, with i a positive integer have been developed. The recursion matrix associated with these recursion relations is pentadiagonal. The inverse of the recursion matrix associated with $i = 2$ is also pentadiagonal. This structure does not necessarily hold for $i > 2$ but a bandwidth structure for the inverse is maintained where the bandwidth increases with an increase in i. We discuss the structure of these inverses and present an application to the computation of rational Gauss quadrature rules.

Joint work with Lothar Reichel (Kent State University, Kent, OH 44242, USA)

A fast algorithm for updating and downsizing the dominant kernel principal components

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Tue 11:25, Room Pacinotti

Many important kernel methods in the machine learning area, such as kernel principal component analysis, feature approximation, denoising, compression and prediction require the computation of the dominant set of eigenvectors of the symmetric kernel Gram matrix. Recently, an efficient incremental approach was presented for the fast calculation of the dominant kernel eigenbasis [1], [2]. In this talk we propose faster algorithms for incrementally updating and downsizing the dominant kernel eigenbasis. These methods are well-suited for large scale problems since they are both efficient in terms of complexity and data management.

[1] G. Gins, I. Y. Smets, and J. F. Van Impe, Efficient tracking of the dominant eigenspace of a normalized kernel matrix, Neural. Comput., 20, (2008), pp. 523–554.
[2] L. Hoegaerts, L. De Lathauwer, I. Goethals, J. A. K. Suykens, J. Vandewalle, and B. De Moor, Efficiently updating and tracking the dominant kernel principal components, Neural Network., 20, (2007), pp. 220–229.

Joint work with Eugene E. Tyrtyshnikov (Russian Academy of Sciences, Moscow, Russia), P. Van Dooren (Catholic University of Louvain, Louvain-la-Neuve, Belgium)

Generalized circulant preconditioners for Toeplitz systems

S. Noschese, SAPIENZA Università di Roma, Italy
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Tue 15:25, Room Pacinotti

A Toeplitz matrix with the first entry of each column obtained by multiplying the last entry of the preceding column by $e^{i\phi}$ is said to be an $\{e^{i\phi}\}$-circulant matrix. In this talk a formula for the distance in the Frobenius norm of a Toeplitz matrix to the set of the $\{e^{i\phi}\}$-circulant matrices is presented. Since the underlying minimization problem generalizes a minimization problem solved by the optimal circulant preconditioner introduced by T. Chan, the natural application of generalized circulants as preconditioners in the PCG method for solving linear systems with a Toeplitz matrix T_n is discussed. Similarly to the circulant case, matrix-vector products $C_n y$ and $C_n^{-1}y$, where C_n is an $\{e^{i\phi}\}$-circulant and y any vector in $\mathbb{C}^n$, can be evaluated in $\mathcal{O}(n \log n)$ arithmetic floating-point operations with the aid of the Fast Fourier Transform. Moreover, the construction of these generalized preconditioners does not require the explicit knowledge of the generating function of T_n, and needs only $\mathcal{O}(n)$ operations. I present theoretical and numerical results that shed light on the performance of these preconditioners. Extensions to generalized circulant preconditioners for two-level Toeplitz systems are also discussed.

Joint work with L. Reichel (Kent State University)

On the power of randomized preconditioning

V. Y. Pan, Lehman College, CUNY, USA
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Wed 12:15, Room Pacinotti

Our randomization techniques turn out to be an important missing ingredient of the known methods of preconditioning and thus dramatically expand their power. For a typical ill conditioned input we perform with a high precision only a small fraction of all flops involved, thus yielding dramatic acceleration of the known algorithms for general and structured input matrices, both in terms of the bit-operation count and the CPU time observed. For Hankel and Toeplitz linear systems of n equations we save the factor $a(n)$ where $a(512) > 15$, $a(1024) > 90$, and $a(2048) > 350$. Our work extends the domain of application for iterative refinement of the solution of a linear system of equations and for Newtons iteration for the inversion of general and structured matrices and enables effective treatment of nearly rank deficient (e.g., nearly singular) general and structured matrices with no pivoting and no orthogonalization. Such matrices regularly appear, e.g., in the Inverse Iteration for eigen-solving, and our approach enables us to incorporate iterative refinement without slowing down the convergence. Further extensions include polynomial root-finding, computation of the numerical rank of a matrix, approximation of a nearly rank deficient matrix with a matrix of a smaller rank, and approximation of a matrix by a nearby Toeplitzlike or Hankel-like matrix. Our 30-minute talk shall outline this 5-year work by presenting the main techniques and some central formal and experimental results, with the pointers to more complete and detailed coverage in our papers in LAA 20092010, in press, and in progress.

Designing a library for structured linear algebra computation.

G. Rodriguez, University of Cagliari, Italy
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Tue 15:00, Room Pacinotti

Two libraries recently developed for computation with structured matrices will be presented. The plans for extending the functionality of these libraries and for integrating them with other available software will be described.

[1] A. Aricò and G. Rodriguez. A fast solver for linear systems with displacement structure. Manuscript.
[2] M. Redivo-Zaglia and G. Rodriguez. `smt`: a Matlab structured matrices toolbox. arXiv:0903.5094 [math.NA], 2009. Submitted.

Joint work with A. Aricò and F. Arrai (University of Cagliari), M. Redivo-Zaglia (University of Padova).

Error Analysis of a Fast Algorithm for Quasiseparable Systems

Michael Stewart, Georgia State University
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Wed 11:00, Room Pacinotti

This work describes a parameterization of an $n \times n$ quasiseparable matrix A in terms of a nested product of small Householder transformations and very sparse bidiagonals. Once computed the parameterization can be exploited for fast, $O(n)$, solution of systems of equations with quasiseparable structure. The represenation is insensitive in the sense that small errors on the parameters correspond to small errors on the matrix. Results of an error analysis show that the algorithm is normwise backward stable.

Joint work with Tom Bella (University of Rhode Island) and Vadim Olshevsky (University of Connecticut)

On structure-preserving Arnoldi-like methods

A. Salam, University Lille Nord de France, France.
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Tue 15:50, Room Pacinotti

In this talk, two structure-preserving Arnoldi-like methods are presented and studied. The obtained methods preserve the structures of a class of structured matrices, including Hamiltonian, skew-Hamiltonian or symplectic matrices. Such methods are useful for computing few eigenvalues and vectors of large and sparse structured matrices. Numerical experiments are given.

[1] A. Bunse-Gerstner and V. Mehrmann, A symplectic QR-like algorithm for the solution of the real algebraic Riccati equation, IEEE Trans. Automat. Control AC-31 (1986), 1104–1113.
[2] A. Salam and E. Al-Aidarous and A. Elfarouk,Optimal symplectic Householder transformations for SR-decomposition, Linear Algebra Appl. 429 (2008), no. 5-6, 1334-1353.
[3] M. Sadkane and A. Salam, A note on symplectic block reflectors, ETNA, Vol. 33 (2009), pp 45-52.

Unmixing of rational functions by tensor computations

M. Van Barel, Katholieke Universiteit Leuven, Belgium
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Tue 12:15, Room Pacinotti

Very recently, Lieven De Lathauwer has introduced various types of Block Term Decompositions (BTD) for higher-order tensors. These decompositions generalize both the Tucker decomposition (or multilinear Singular Value Decomposition) and the Canonical / Parallel Factor decomposition. The latter are related with low multilinear rank approximation and low rank approximation of higher-order tensors, respectively. It turns out that BTDs can be used for source separation and factor analysis. In this talk, we investigate the possibility of unmixing rational functions using a particular type of BTD. We show the effectiveness of using the BTD by some numerical examples.

Joint work with L. De Lathauwer (Katholieke Universiteit Leuven, Belgium)

A multishift QR-algorithm for hermitian plus low rank matrices

R. Vandebril, K.U.Leuven, Belgium
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Wed 11:50, Room Pacinotti

Hermitian plus possibly non-Hermitian low rank matrices can be efficiently reduced into Hessenberg form. The resulting Hessenberg matrix can still be written as the sum of a Hermitian plus low rank matrix.

In this talk we will discuss a new implicit multishift QR-algorithm for Hessenberg matrices, which are the sum of a Hermitian plus a possibly non-Hermitian low rank correction.

The proposed algorithm exploits both the symmetry and low rank structure to obtain a QR-step involving only $O(n)$ floating point operations instead of the standard $O(n^2)$ operations needed for performing a QR-step on a Hessenberg matrix. The algorithm is based on a suitable $O(n)$ representation of the Hessenberg matrix. The low rank parts present in both the Hermitian and low rank part of the sum are compactly stored by a sequence of Givens transformations and few vectors.

Due to the new representation, we cannot apply classical deflation techniques for Hessenberg matrices. A new, efficient technique is developed to overcome this problem.

Some numerical experiments based on matrices arising in applications are performed. The experiments illustrate effectiveness and accuracy of both the QR-algorithm and the newly developed deflation technique.

Joint work with G. M. Del Corso (University of Pisa)

Twisted Green's (CMV-like) matrices and their factorizations, Laurent polynomials and Digital Filter Structures.

P. Zhlobich, University of Connecticut, USA
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Tue 17:10, Room Pacinotti

Several new classes of structured matrices have appeared recently in the scientific literature. Among them there are so-called CMV and Fiedler matrices which are found to be related to polynomials orthogonal on the unit circle and Horner polynomials, respectively. Both matrices are five diagonal and have a similar structure, although they have appeared under completely different circumstances.

In a recent paper by Bella, Olshevsky and Zhlobich, it was proposed a unified approach to the above mentioned matrices. Namely, it was shown that all of them belong to a wider class of twisted Green's matrices. We will use this idea to show that the factorizability of CMV and Fiedler matrices into a product of planar rotations in the n-dimensional space is also inherited by twisted Green's matrices. Shortly, for a given Hessenberg Green's matrix of size n, the interchange of factors in the factorization leads to 2^n different twisted Green's matrices.

CMV matrix appeared in the scientific literature in connection with Laurent polynomials orthogonal on the unit circle. Fiedler matrix was developed purely from its factorization. We will show that an infinite-dimensional twisted Green's matrix serve as the operator of multiplication by "z" in the linear space of complex Laurent polynomials. Our development doesn't use orthogonallity in any sense and is based on the factorization and recurrence relations only. In the case of finite dimensional matrices we are able to give an explicit form of an eigenvector and all the generalized eigenvectors for a given eigenvalue.

The final part of our talk will be devoted to Kimura's approach to CMV matrices, i.e. Signal Flow Graphs (SFG) approach. We will exploit the tool of SFG to visualize all the theoretical results for twisted Green's matrices as well as to show how they can be used in construction of new types of Digital Filters.

Joint work with F. Marcellán (Univ. Carlos III de Madrid), V. Olshevsky (Univ. of Connecticut) and G. Strang (MIT)

Markov Chains

Steve Kirkland, National University of Ireland, Maynooth
Michael Neumann, University of Connecticut, USA

On the mean first passage matrix of a simple random walk on a tree

R.B. Bapat, Indian Statistical Institute New Delhi, India
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Mon 17:35, Room B

We consider a simple random walk on a tree. Exact expressions are obtained for the expectation and the variance of the first passage time, thereby recovering the known result that these are integers. A relationship of the mean first passage matrix with the distance matrix is established and used to derive a formula for the inverse of the mean first passage matrix.

Probabilistic Approach to Perron Root, the Group Inverse, and Applications

Iddo Ben Ari, University of Connecticut, Storrs, USA

Mon 17:10, Room B

A probabilistic approach to the study of the Perron root of irreducible nonnegative matrices is presented. This is then applied to reestablish and improve some known results in the field. The analysis focuses on perturbative theory for the Perron root the group inverse of a generator of a continuous time Markov chain, and their relations.

Restricted additive Schwarz methods for computing the stationary vector of large Markov chains

Michele Benzi, Emory University, Atlanta, GA, USA
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Wed 12:15, Room B

The Restricted Additive Schwarz (RAS) algorithm is a domain decomposition method that has proved very effective in solving large sparse systems of linear equations arising from the discretization of partial differential equations on parallel computers. In this talk we extend the RAS algorithm to the problem of computing the steady-state (stationary) vector of Markov chains with large state spaces. We prove convergence of the stationary iterative method, and we address computational issues such as partitioning, the amount of overlap, inexact subdomain solves, the construction of two-level schemes bases on "coarse grid" corrections, and Krylov subspace acceleration. The results of numerical experiments on matrices arising from real applications in Markov modelling will be presented.

Joint work with Verena Kuhlemann (Emory University)

Coupling and Mixing in Markov Chains

Jeffrey J Hunter, Auckland University of Technology & Masey University, New Zealand

Wed 11:25, Room B

Following a discussion of the concepts of mixing and coupling in Markov chains, expressions for the expected times to mixing and coupling are developed. The two-state cases and three-state cases are examined in detail and some results for the bounds on the expected values are given. The key results are given in Hunter, J.J.: "Coupling and mixing times in a Markov chain", Linear Algebra and its Applications, 430, 2607-2621, (2009), and Hunter, J.J.: "Bounds on Expected

Coupling Times in a Markov Chain", (pp271-294), "Statistical Inference, Econometric Analysis and Matrix Algebra. Festschrift in Honour of Gtz Trenkler", Bernhard Schipp and Walter Kraemer (Editors), Physica-Verlag Heidelberg (2009)

To be Announced

Ali Jadbabaie, University of Sydney, Australia

Wed 11:50, Room B

Non-negative matrix products and John Hajnal (1924-2008)

Eugene Seneta, School of Mathematics and Statistics, FO7, University of Sydney, Australia

Mon 16:45, Room B

Hajnal (Proc. Cambridge Philos. Soc., 54 (1958), 233-246) gave the first proof of a necessary and sufficient condition for weak ergodicity of a sequence of nxn stochastic matrices (non-negative matrices with row sums one). Intrinsically, he used the Markov-Dobrushin coefficient of ergodicity, whose sub-unit value indicates contractivity for a stochastic matrix.

He also introduced the key idea of a scrambling matrix. Later (Math. Proc. Cambridge Phil. Soc., 79(1976)521-530), motivated by work in demography on inhomogeneous products of certain kinds of non-stochastic non-negative matrices, he developed a weak ergodicity theory for general nonnegative matrix products, using Birkhoff's contraction ratio. Both papers were enormously influential on subsequent developments.

This tribute to Hajnal outlines his biography, motivation and methodology, and briefly synthesizes the history of inhomogeneous products pre- and post- 1957, including the work of Doeblin and Sarymsakov.

The Inverse Mean First Passage Matrix Problem And The Inverse M-Matrix Problem

Raymond Nung-Sing Sze, Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Hong Kong
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Wed 11:00, Room B

The inverse mean first passage time problem is given an $n \times n$ positive matrix M, then when does there exist an n state discrete time homogeneous ergodic Markov chain, whose mean first passage matrix is M. The inverse M-matrix problem is given a nonnegative matrix A, then when is A an inverse of an M-matrix. In this talk, results concerning these two problems are discussed.

Joint work with M. Neumann, Univesrity of Connecticut

Nonnegative Matrices

Judi McDonald
Michael Tsatsomeros

Nonnegative matrix theory is an important area of linear algebra that has been built up from the Perron-Frobenius Theorem and has largely been driven by applications. This minisymposium brings together individuals with experience and interests in classical nonnegative matrix theory, as well as in a variety of generalizations and applications.

Spectrally arbitrary patterns over finite fields.

E. J. Bodine, Washington State University
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Fri 17:35, Room Pacinotti

In this talk, we will explore zero-nonzero patterns over finite fields. In particular, we will examine patterns that demonstrate fundamental differences in the algebraic structure of different fields.

Joint work with J. J. McDonald (Washington State University)

Sign patterns that require or allow power-positivity

Minerva Catral, Iowa State University, USA
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Fri 16:45, Room Pacinotti

A matrix A is power-positive if some positive integer power of A is entrywise positive. A matrix A is eventually positive if A^k is entrywise positive for all sufficiently large integers k. A characterization of sign patterns that require power-positivity is presented. It is also shown that a sign pattern $\mathcal{A}$ allows power-positivity if and only if $\mathcal{A}$ or $-\mathcal{A}$ allows eventual positivity.

Joint work with Leslie Hogben, Iowa State University, USA (lhogben@iastate.edu) & American Institute of Mathematics (hogben@aimath.org), D. D. Olesky, University of Victoria, Canada (dolesky@cs.uvic.ca), P. van den Driessche, University of Victoria, Canada (pvdd@math.uvic.ca)

Matrix Functions Preserving Sets of Generalized Nonnegative Matrices

A. Elhashash, Drexel University, USA
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Thu 12:15, Room Pacinotti

We characterize matrix functions preserving several sets of generalized nonnegative matrices. These sets include PFn, the set of $n \times n$ real eventually positive matrices; and WPFn, the set of matrices $A \in \mathbb{R}^{n\times n}$ such that A and its transpose has the Perron-Frobenius property. We also present necessary and sufficient conditions for a matrix function to preserve the set of $n\times n$ real eventually nonnegative matrices and the set of $n\times n$ real exponentially nonnegative. Moreover, we show that the only complex polynomials that preserve the set of $n \times n$ real exponentially nonnegative matrices are $p(z) = az + b$ where $a, b \in \mathbb{R}$ and $a \geq 0$.

Joint work with D. B. Szyld (Temple University)

Nonnegative Jordan bases and characterization of eventually nonnegative matrices

D. Noutsos, University of Ioannina, Greece
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Thu 11:00, Room Pacinotti

For an $n \times n$ eventually nonnegative matrix A, the existence of a nonnegative Jordan basis of the eigenspace corresponding to the dominant eigenvalue is proven. This result is used to characterize when the matrix $hI + A$ is eventually nonnegative for all $h > 0$. Sufficient and necessary conditions are proven for this situation. Numerical examples are presented to illustrate and validate the theoretical results.

Joint work with M. J. Tsatsomeros (Washington State University)

Sign Patterns that Allow Eventual Positivity
Dale Olesky, University of Victoria, Victoria, BC Canada
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Fri 15:50, Room Pacinotti

A real square matrix A is *eventually positive* if there exists a positive integer k_0 such that for all $k \geq k_0$, A^k is entrywise positive. Some necessary or sufficient conditions for a sign pattern to allow eventual positivity are given. It is shown that certain families of sign patterns do not allow eventual positivity, and that for $n \geq 2$, the minimum number of positive entries in an $n \times n$ sign pattern that allows eventual positivity is $n + 1$. All 2×2 and 3×3 sign patterns are classified as to whether or not the pattern allows eventual positivity. A 3×3 matrix is presented to demonstrate that the positive part of an eventually positive matrix need not be primitive, answering negatively a question of Johnson and Tarazaga.

Computations with totally nonnegative and sign-regular matrices
J.M. Peña, University of Zaragoza, Spain
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Thu 11:25, Room Pacinotti

Totally nonnegative matrices are matrices with all their minors nonnegatives. They belong to the class of sign-regular matrices. We present recent advances of numerical methods for these classes of matrices. Methods for solving linear systems and methods concerning with their factorizations and related accurate computations are considered, as well as tests to recognize if a matrix belongs to these classes of matrices. Recent advances on localization results for their eigenvalues are also analyzed.

Some constructive methods in the symmetric nonnegative inverse eigenvalue problem
H. Šmigoc, University College Dublin, Dublin
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Fri 15:00, Room Pacinotti

The question, which lists of complex numbers are the spectrum of some nonnegative matrix, is known as the nonnegative inverse eigenvalue problem (NIEP), and the same question posed for symmetric nonnegative matrices is called the symmetric nonnegative inverse eigenvalue problem (SNIEP). In the talk we will present some constructive methods applied to the SNIEP and discuss the effect of adding zeros to the spectrum in the SNIEP.

Joint work with T. J. Laffey(University College Dublin)

Graphs, Patterns and Powers – From Nonnegative Matrices to Nonpowerful Ray Patterns
Jeffrey Stuart, Pacific Lutheran University, Tacoma, Washington, USA
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Fri 15:25, Room Pacinotti

For positive powers of square matrices, which properties arise from the signed digraph and which depend on the relative magnitudes of the entries? There is a substantial body of research on this question, starting with the work on primitivity and imprimitivity for nonnegative matrices. Later, this study was extended to examining the zero-nonzero patterns of powers of real matrices and their connection to the signed digraph for the matrix. Viewed in terms of sign pattern classes, this led to the study of powerful sign patterns as well as the base and period of a powerful sign pattern. Subsequently, the results for powerful sign pattern classes were extended to results for powerful ray pattern classes, although there are some interesting differences between the behavior of sign patterns and ray patterns. Recently, interest has shifted to the behavior of powers of square, complex ray patterns that are not powerful, which is to say, for which the ray-signed digraph does not determine the ray patterns of powers of matrices in the qualitative class. For nonpowerful ray patterns, the concepts of base, power and primitivity break down in ways that have sometimes been overlooked in recent papers. We will discuss these ideas and some of the current directions of research.

On the periodic stabilization of discrete-time positive switched systems
Maria Elena Valcher, University of Padova, Italy
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Thu 11:50, Room Pacinotti

Positive switched systems typically arise to cope with two distinct modeling needs. On the one hand, switching among different models mathematically formalizes the fact that the system laws change under different operating conditions. On the other hand, the variables to be modeled may be quantities that have no meaning unless positive (temperatures, pressures, population levels, ...). Research interests in positive switched systems mainly focused on their stability properties [1], while stabilizability has been only marginally touched upon. In this talk we present some results about the stabilization of positive switched systems. In detail, we consider the class of discrete-time positive switched systems, described, at each time $t \in \mathbb{Z}_+$, by the first-order difference equation:

$$x(t+1) = A_{\sigma(t)}x(t), \quad (1)$$

where $x(t)$ denotes the n-dimensional state variable at time t, while σ is a switching sequence, defined on $\mathbb{Z}_+$ and taking values in the finite set $\mathcal{P} = \{1, 2\}$. For each $i \in \mathcal{P}$, A_i is an $n \times n$ positive matrix.

Assuming that both A_1 and A_2 are not Schur matrices, we provide conditions on the matrix pair (A_1, A_2) that ensure that for every positive initial state $x(0) \in \mathbb{R}^n_+$, there is a switching law $\sigma : \mathbb{Z}_+ \to \mathcal{P}$ that leads the state trajectory $x(t) = A_{\sigma(t-1)}A_{\sigma(t-2)} \dots A_{\sigma(1)}A_{\sigma(0)}x(0)$ to zero as t goes to $+\infty$. In particular, we show that every stabilizable positive switched system can be stabilized by means of a periodic switching law. Finally, we prove that, if a Schur convex combination of the matrices A_1 and A_2 can be found, it provides some information on the period and the "duty cycle" of these stabilizing periodic switching sequences.

[1] O. Mason and R.N. Shorten. Quadratic and copostive Lyapunov functions and the stability of positive switched linear systems. In Proceedings of the American Control Conference (ACC 2007), pp. 657–662, New York, 2007.

Joint work with Ettore Fornasini (University of Padova)

Spectrally arbitrary zero-nonzero patterns
A. A. Yielding, Eastern Oregon University, La Grande, OR

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Fri 17:10, Room Pacinotti

In this talk we establish the lower bound of $2n-2$ for the number of zero entries an $n \times n$ irreducible zero-nonzero pattern that is not spectrally arbitrary and contains at least two nonzero entries along the diagonal.

Joint work with J. J. McDonald (Washington State University)

Matrix Functions and Matrix Equations

Chun Hua Guo, University of Regina, Canada
Valeria Simoncini, University of Bologna, Italy

A Newton-Galerkin-ADI Method for Large-Scale Algebraic Riccati Equations

Peter Benner, TU Chemnitz, Germany
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Thu 16:45, Room Pacinotti

Solving large-scale algebraic Riccati equations (AREs) is one of the central tasks in solving optimal control problems for linear and, using receding-horizon techniques, also nonlinear instationary partial differential equations. Large-scale AREs also occur in several model reduction methods for dynamical systems. Due to sparsity and large dimensions of the resulting coefficient matrices, standard eigensolver-based methods for AREs are not applicable in this context. In the recent two decades, several approaches for such large-scale AREs have been suggested. They mainly fall into two categories:

1. *Galerkin-projection*: the ARE is projected onto a low-dimensional subspace, e.g., a suitable Krylov subspace, then the small scale ARE is solved using a standard solver and the solution is prolongated to full-scale;
2. *Newton's method:* exploit sparsity in the resulting linear system of equations (= a Lyapunov equation) to be solved in each step.

Here, we will present the hybrid method suggested in [1]. It is based on exploiting the advantages of both ideas. Numerical experiments confirm the high efficiency of this new method and demonstrate its applicability to the aforementioned application areas.

[1] P. Benner and J. Saak, A Galerkin-Newton-ADI Method for Solving Large-Scale Algebraic Riccati Equations. Preprint SPP1253-090, DFG Priority Programme 1253 "Optimization with Partial Differential Equations", January 2010.

Joint work with Jens Saak (TU Chemnitz)

Computation of matrix functions arising in the analysis of complex networks

Michele Benzi, Emory University, Atlanta, GA, USA
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Thu 15:00, Room Pacinotti

Quantitative methods of network analysis naturally lead to large-scale computations for functions of matrices associated with sparse graphs. This talk will describe some of the main quantities of interest in network analysis as introduced by Estrada, Hatano, D. Higham and others. We combine decay bounds [2,3] and Gaussian quadrature rules [4] to derive a priori bounds and efficient numerical methods for estimating the quantities of interest. Numerical experiments using small-world, range-free, and Erdös-Renyi graphs will be used to illustrate the algorithms. This talk is based in part on the report [1].

[1] M. Benzi and P. Boito, Quadrature Rule-Based Bounds for Functions of Adjacency Matrices, Technical Report TR-2009-031, Department of Mathematics and Computer Science, Emory University, January 2010.
[2] M. Benzi and G. H. Golub, Bounds for the entries of matrix functions with applications to preconditioning, BIT, 29 (1999), pp. 417–438.
[3] M. Benzi and N. Razouk, Decay bounds and $O(n)$ algorithms for approximating functions of sparse matrices, ETNA, 28 (2007), pp. 16–39.
[4] G. Meurant and G. H. Golub, Matrices, Moments and Quadrature with Applications. Princeton University Press, Princeton, NJ, 2010.

Joint work with Paola Boito (Emory University and CERFACS)

On the numerical solution of the matrix equation $\sum_{i=1}^{k} \log(XA_i^{-1}) = 0$

D.A. Bini, University of Pisa, Italy
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Mon 15:00, Room Pacinotti

Let $A_i,\ i = 1, \ldots, k$ be real symmetric positive definite $n \times n$ matrices. It is known that the minimum of the function $\sum_{i=1}^{k} d(X, A_i)^2$ for $d(X, Y) = ||X^{-1/2}YX^{-1/2}||_F$ is attained at a matrix X which solves the equation $\sum_{i=1}^{k} \log(XA_i^{-1}) = 0$. This solution X is called the Karcher mean of the matrices $A_1, \ldots, A_k$.

We introduce the iteration

$$X_{\nu+1} = X_\nu \exp(\theta \sum_{i=1}^{k} \log(XA_i))$$

and its first order approximation

$$X_{\nu+1} = X_\nu + \theta X_\nu \sum_{i=1}^{k} \log(XA_i)$$

for approximating the Karcher mean.

We provide a convergence analysis with a dynamical determination of the optimal parameter θ and show that under certain conditions, convergence is locally quadratic with the optimal choice of θ. We provide a way for the choice of an initial approximation which greatly speeds up the convergence. Numerical experiments which validate our analysis are reported.

Joint work with B. Iannazzo (University of Perugia)

On different classes of Lyapunov equations

Tobias Damm, University of Kaiserslautern, Germany
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Mon 15:25, Room Pacinotti

Lyapunov equations are fundamental e.g. in stability analysis or model order reduction reduction. As is well-known, different forms of Lyapunov operators occur for different classes of systems such as linear stochastic systems, linear delay equations or bilinear systems. We give a short review of these matrix equations and report on some new results and numerical methods.

Inertia and Rank Characterizations of the Expressions $A - BXB^* - CYC^*$ and $A - BXC^* \pm CX^*B^*$

Delin Chu, National University of Singapore, Singapore

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Mon 15:50, Room Pacinotti

In this paper we consider the admissible inertias and ranks of the expressions $A-BXB^*-CYC^*$ and $A-BXC^*\pm CX^*B^*$ with unknowns X and Y in the four cases when these expressions are : (i) complex self-adjoint,(ii) complex skew-adjoint, (iii) real symmetric, (iv) real skew symmetric. We also provide a construction for X and Y to achieve the desired inertia/rank, that uses only unitary/orthogonal transformation thus leading to a numerically reliable construction. Consequently, necessary and sufficient solvability conditions for the matrix equations

$$A-BXB^*-CYC^*=0,$$

and

$$A-BXC^*\pm CX^*B^*=0$$

are provided.

Joint work with (Y.S.Hung (Department of Electrical and Electronic Engineering. The University of Hong Kong, Hong Kong) and Hugo J. Woerdeman (Department of Mathematics, Drexel University, Philadelphia, USA.))

Hierarchical and Multigrid Methods for Matrix and Tensor Equations

L. Grasedyck, RWTH Aachen, Germany
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Mon 11:00, Room Pacinotti

Hierarchical and Multigrid methods are among the most efficient methods for the solution of large-scale systems that stem, e.g. from the discretization of partial differential equations (PDE). In this talk we will review the generalization of these methods to the solution of matrix equations [1], [2], and equations that possess a tensor structure [3]. The standard hierarchical and multigrid methods can perfectly be combined with low rank (matrix) and low tensor rank representations. The benefit is that the solution is computable in almost optimal complexity with respect to the amount of data needed for the representation of the solution. As an example we consider a PDE posed in a product domain $\Omega\times\Omega, \Omega\subset\mathbb{R}^d$ and discretized with N^d basis functions for the domain Ω. Under separability assumptions on the right-hand side the system is solved in low rank form in $\mathcal{O}(N^d)$ complexity (instead of $\mathcal{O}(N^{2d})$ required for the full solution). For a PDE on the product domain $\underbrace{\Omega\times\cdots\times\Omega}_{D\text{ times}}$ one can even solve the system in low tensor rank form in $\mathcal{O}(N^d)$ complexity (instead of $\mathcal{O}(N^{Dd})$ required for the full solution). The state of the art will be shortly summarized.

[1] L. Grasedyck, W. Hackbusch, A Multigrid Method to Solve Large Scale Sylvester Equations, SIMAX 29, pp. 870-894, 2007.
[2] L. Grasedyck, Nonlinear multigrid for the solution of large scale Riccati equations in low-rank and H-matrix format, Num.lin.alg.appl. 15, pp. 779-807, 2008.
[3] L. Grasedyck, Hierarchical Singular Value Decomposition of Tensors, Technical Report 27/2009, Max Planck Institute for Mathematics in the Sciences, Leipzig, `www.mis.mpg.de`.

Krylov-enhanced parallel integrators for linear problems

S. Güttel, University of Geneva, Switzerland
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Thu 15:50, Room Pacinotti

The parareal algorithm is a numerical method to integrate evolution problems on parallel computers. The main components of this algorithm are a coarse integrator, which quickly propagates information on a coarse partition of the time interval, and a fine integrator, which solves the evolution problems more accurately on each subinterval. The performance of this algorithm is well understood for diffusive problems, but it can also have spectacular performance when applied to certain non-linear problems. In [2] the authors proposed a Krylov-enhanced version of the parareal algorithm, which for linear problems is equivalent to the modified PITA algorithm described in [1]. Both of these algorithms can be successful for 2nd order ODE's. Refining the analysis in [2], we study the convergence of the Krylov-enhanced parareal algorithm and consider the particularly interesting special case when the coarse integrator is a polynomial or rational Krylov-based exponential or trigonometric integrator.

[1] C. Farhat, J. Cortial, C. Dastillung & H. Bavestrello, Time-parallel implicit integrators for the near-real-time prediction of linear structural dynamic responses. Internat. J. Numer. Methods Engrg. 67 (2006), pp. 697–724.
[2] M. Gander & M. Petcu, Analysis of a Krylov subspace enhanced parareal algorithm for linear problems. ESAIM: Proc. 25 (2008), pp. 114–129.

Rational Approximation to Trigonometric Operators

M. Hochbruck, Karlsruhe Institute of Technology, Germany

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Thu 15:25, Room Pacinotti

We will discuss the approximation of trigonometric operator functions that arise in the numerical solution of wave equations by trigonometric integrators. It is well known that Krylov subspace methods for matrix functions without exponential decay show superlinear convergence behavior if the number of steps is larger than the norm of the operator. Thus, Krylov approximations may fail to converge for unbounded operators. In this talk, a rational Krylov subspace method is proposed which converges not only for finite element or finite difference approximations to differential operators but even for abstract, unbounded operators. In contrast to standard Krylov methods, the convergence will be independent of the norm of the operator and thus of its spatial discretization. We will discuss efficient implementations for finite element discretizations and illustrate our analysis with numerical experiments.

[1] V. Grimm und M. Hochbruck Rational approximation to trigonometric operators BIT, vol. 48, no. 2, pp. 215229 (2008)

Joint work with V. Grimm (Karlsruhe Institute of Technology)

A binary powering Schur algorithm for computing primary matrix roots

B. Iannazzo , Università di Perugia, Italy
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Thu 17:35, Room Pacinotti

Let p be a positive integer. A primary pth root of a square matrix A is a solution of the matrix equation $X^p-A=0$ which can be written as a polynomial of A.

If A has no nonpositive real eigenvalues then there exists only one primary pth root whose eigenvalues lie in the sector

$\mathcal{S}_p = \{z \in \mathbb{C} \setminus \{0\} : |\arg(z)| < \pi/p\}$, which is called principal pth root and denoted by $A^{1/p}$.

The main numerical problem is to compute $(A^{1/p})^r$, for $0 < r < p$ integer. This problem is encountered in certain applications, among which financial models, and in the numerical computation of other matrix functions [2].

We present an algorithm for computing primary roots of a nonsingular matrix A. The algorithm is based on the Schur decomposition of A. In particular, if A has no nonpositive real eigenvalues, it computes $A^{1/p}$ using only real arithmetics.

The algorithm has an order of complexity lower than the customary Schur based algorithm, namely the Smith algorithm [3], and it is a valid alternative to the algorithms based on rational matrix iterations.

[1] F. Greco and B. Iannazzo, *A binary powering Schur algorithm for computing primary matrix roots*, Numer. Algorithms, 2010.
[2] N. J. Higham, *Functions of Matrices: Theory and Computation*, SIAM, Philadelphia, USA, 2008.
[3] M. I. Smith, *A Schur algorithm for computing matrix pth roots*, SIAM J. Matrix Anal. Appl., 2003.

Joint work with F. Greco (Università di Perugia)

Error estimates for two rational Krylov subspace methods to solve the Lyapunov equation with a rank one right-hand side

L. Knizhnerman, Mathematical Modelling Department of Central Geophysical Expedition, Moscow, Russia
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Mon 12:15, Room Pacinotti

The Extended Krylov Subspace Method has recently arisen as a competitive method for solving large-scale Lyapunov equations. Using the theoretical framework of orthogonal rational functions (Faber–Dzhrbashyan series, Blaschke products), in this talk we report on a general a priori error estimate when the known term has rank one, i.e., the equation has the form

$$AX + XA^* + bb^* = 0, \qquad A, X \in \mathbf{R}^{N\times N}, \qquad b \in \mathbf{R}^N,$$

with a positively definite known matrix A.

We also apply the same technique to analyze the behavior of the Rational Krylov Subspace Method, applied to the same problem, with a priori chosen shifts (EKSM corresponds to cyclically repeated shifts 0 and ∞).

Special cases, such as symmetric coefficient matrix, are also treated.

Numerical experiments confirm the proved theoretical assertions.

Joint work with V. Druskin (Schlumberger–Doll Research, Cambridge, USA), V. Simoncini (University of Bologna, Italy), M. Zaslavsky (Schlumberger–Doll Research, Cambridge, USA)

Filters connecting quadratic systems

Peter Lancaster, University of Calgary, Canada.
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Mon 16:45, Room Pacinotti

The diagonalization of quadratic systems $L(\lambda) = M\lambda^2 + D\lambda + K$ is a fundamental problem in many applications. These systems may have real or complex matrix coefficients, with or without symmetries. Diagonalization by the application of strict equivalence or congruence transformations directly to $L(\lambda)$ is well-understood but is possible for only a very restrictive class of systems. Diagonalization by applying structure preserving transformations to a *linearization* of $L(\lambda)$ has also been developed recently, and is possible for a wider class of systems.

Here, we describe the possibility of finding *linear* systems of the form $F(\lambda) := F_1\lambda + F_0$ for which

$$\tilde{F}(\lambda)L(\lambda) = \tilde{L}(\lambda)F(\lambda)$$

and $\tilde{L}(\lambda)$ is *diagonal.* We call thesefunctions *linear filters.*

We show how filters can be constructed using familiar structures of "standard pairs" and "structure preserving transformations".

Joint work with S.D.Garvey, (University of Nottingham, UK), A.Popov, (University of Nottingham, UK), U.Prells, (University of Nottingham, UK), I.Zaballa, (Euskal Herriko Unibersitatea, Spain).

Stabilizing complex symmetric solution of the equation $X + A^\top X^{-1}A = Q$ arising in nano research

Wen-Wei Lin, National Chiao Tung University, Taiwan
wwlin@math.nctu.edu.tw
Mon 17:10, Room Pacinotti

We study the existence and characteristic of the stabilizing complex symmetric solution X_s for the matrix equation $X + A^\top X^{-1}A = Q$ arising in nano research. In stead of using the deep theory of linear operators we give a new proof on the existence of X_s by using only basic knowledge of linear algebra. Furthermore, we show that the imaginary part of X_s is positive semi-definite with rank=$m/2$, where m is the number of simple unimodular eigenvalues of the rational matrix-valued function $\psi(\lambda) \equiv Q + \lambda A + \lambda^{-1}A^\top$. We also present a doubling algorithm for computing the desired solution X_s efficiently and reliably.

Joint work with Chun-Hua Guo (University of Regina, Canada) and Yueh-Cheng Kuo (National University of Kaohsiung, Taiwan).

Algorithms for nonnegative quadratic vector equations

F. Poloni, Scuola Normale Superiore, Pisa, Italy
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Mon 17:35, Room Pacinotti

We investigate a vector equation having the form

$$Mx = a + b(x, x), \tag{1}$$

where $a, x \in \mathbb{R}^n_{\geq 0}$, M is an $n\times n$ M-matrix and $b : \mathbb{R}^n_{\geq 0} \times \mathbb{R}^n_{\geq 0} \to \mathbb{R}^n_{\geq 0}$ is a bilinear map. The equation (1) appears in the study of Markovian binary trees [Bean, Kontoleon Taylor, *Ann. Oper. Res.* '08; Hautphenne, Latouche, Remiche, *LAA* '08].

We propose a new functional iteration (and a corresponding Newton method) for its solution, based on the computation of the Perron vector of a special matrix. The most interesting property of these methods is that their convergence behaviour does not degrade when the equation is close to null recurrent, in contrast to the traditional algorithms. This means that they are particularly effective on the most "difficult" problems.

Moreover, we may weaken the hypotheses of the original probabilistic equation in order to obtain a general framework for systems of quadratic equations with nonnegativity constraints, encompassing nonsymmetric algebraic Riccati equations [Guo, Laub, *SIMAX* '00], Lu's simple equation [Lu L.-Z., *SIMAX* '05], and several quadratic equations in queuing theory and probability [Bini, Latouche, Meini, *LAA* '02 and '03]. This allows us to give a unified treatment of the numerical methods for their solution. In some cases, this unification leads to new algorithms or more general proofs.

It is still an open problem whether it is possible to extend the new Perron vector-based iterations to this larger family of quadratic equations.

Joint work with D. A. Bini (University of Pisa), B. Meini (University of Pisa)

Lur'e Equations and Even Matrix Pencils

T. Reis, TU Berlin / TU Hamburg-Harburg (Germany)
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Mon 11:25, Room Pacinotti

Lur'e equations are a generalization of algebraic Riccati equations and they arise in linear-quadratic optimal control problem which are singular in the input. It is well-known that there is a one-to-one correspondence between the solutions of Riccati equations and Lagrangian eigenspaces of a certain Hamiltonian matrix. The aim of this talk is to generalize this concept to Lur'e equations. We are led to the consideration of deflating subspaces of even matrix pencils.

Dimension reduction for damping optimization of linear vibrating systems

Ninoslav Truhar, University of Osijek, Croatia
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Mon 11:50, Room Pacinotti

Consider a damped linear vibrational system described by the differential equation

$$M\ddot{x} + D\dot{x} + Kx = 0\,, \qquad x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0\,,$$

where M, D, K are mass, damping and stiffness matrix, respectively.

A very important question arises in considerations of such systems: *for given mass and stiffness determine the damping matrix so as to insure an optimal evanescence.*

It can be shown that this optimization problem is equivalent to the following minimization problem:

$$\mathrm{trace}(X) = \min\,,$$

where X is solution of the following Lyapunov equation:

$$AX + XA^T = -GG^T\,,$$

here A is $2n \times 2n$ matrix obtained from M, D and K, and G is matrix with full column rank, and $\mathrm{rank(G)} \ll n$.

Finding the optimal D such that the trace of X is minimal is a very demanding problem, caused by the large number of trace calculations, which are required for bigger matrix dimensions. We propose a dimension reduction to accelerate the optimization process and we present corresponding error bound for the approximation of the solution of Lyapunov equation obtained by this reduction. We will show a new estimates for the eigenvalue decay of the solution X which include the influence of the right-hand side G on the eigenvalue decay rate of the solution. Also, we will present an efficient algorithm for the minimization of $\mathrm{trace}(X)$ using a low rank Cholesky ADI method based on a new set of ADI parameters.

Joint work with Peter Benner, Chemnitz University of Technology, Germany and Zoran Tomljanović, University of Osijek, Croatia

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Algorithms for matrix functions

K. Zię tak, Wrocław University of Technology, Poland
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Thu 17:10, Room Pacinotti

The matrix sector function, introduced by Shieh, Tsay and Wang, is a generalization of the matrix sign function. For a positive integer p and a matrix $A \in \mathbb{C}^{n\times n}$, having no eigenvalues with argument $(2k+1)\pi/p$ for $k = 0, 1, \ldots, p-1$, the matrix sector function is defined by $\mathrm{sect}_p\,(A) = A(\sqrt[p]{A^p})^{-1}$, where $\sqrt[p]{X}$ denotes the principal pth root of X. For $p = 2$ the matrix sector function is the matrix sign function.

We derive and investigate a family of iterations for the sector function, based on the Padé approximants of a certain hypergeometric function. This generalizes a result of Kenney and Laub [3] for the sign function and yields a whole family of iterative methods for computing the matrix pth root.

We prove that the principal Padé iterations for the matrix sector function are structure preserving. It generalizes the result of Higham, Mackey, Mackey, Tisseur [1] for the principal Padé iterations for the matrix sign function (see also Iannazzo [2]).

We also focus on the coupled Padé iterations for computing the matrix pth root. The talk is based on [4] and some current investigations.

[1] N.J. Higham, D.S. Mackey, N. Mackey, F. Tisseur, Computing the polar decomposition and the matrix sign decomposition in matrix groups, *SIAM J. Matrix Anal. Appl.* 25 (2004), 1178–1192.

[2] B. Iannazzo, A family of rational iterations and its application to the computation of the matrix pth root, *SIAM J. Matrix Anal. Appl.*, 30 (2008), 1445–1462.

[3] Ch.S. Kenney, A.J. Laub, Rational iterative methods for the matrix sign function, *SIAM J. Matrix Anal. Appl.* 12 (1991), 273–291.

[4] B. Laszkiewicz, K. Ziętak, A Padé family of iterations for the matrix sector function and the matrix pth root, *Numer. Lin. Alg. Appl.* 16 (2009), 951–970.

Combinatorial Linear Algebra

Shaun Fallat, University of Regina, Canada
Bryan Shader,University of Wyoming, USA

This minisymposium will highlight recent advances in the use of linear algebra to reveal the intrinsic combinatorial structure of matrices described by graphs and digraphs; and the use of graph theory in developing deeper algebraic and analytic theory for matrices that incorporates the underlying structure of the matrix.

The spectral radius and the diameter of connected graphs

Sebastian M. Cioabă, University of Delaware
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Mon 11:00, Room C

In this talk, I will discuss the problem of determining the minimum spectral radius of order n and diameter D. I will focus on the cases when D is constant and when D grows linearly with n.

[1] S.M. Cioabă, E. van Dam, J. Koolen and J.H. Lee, Asymptotic results on the spectral radius and the diameter of graphs, *Linear Algebra and its Applications*, **432** 722-737, (2010).
[2] S.M. Cioabă, E. van Dam, J. Koolen and J.H. Lee, A lower bound for the spectral radius of graphs with fixed diameter, *European Journal of Combinatorics*, to appear.

Joint work with Edwin van Dam (Tilburg University), Jack Koolen (POSTECH) and Jae-Hoo Lee (University of Wisconsin-Madison)

Majorization permutahedra and $(0,1)$-matrices
G. Dahl, University of Oslo, Norway
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Tue 11:50, Room C

Let $x_{[j]}$ denote the jth largest component of a real vector x. For vectors $x, v \in \Re^n$ one says that x is *majorized* by v ([1], [2]), denoted by $x \preceq v$, provided that $\sum_{j=1}^{k} x_{[j]} \leq \sum_{j=1}^{k} v_{[j]}$ for $k = 1, \ldots, n$ where there is equality for $k = n$. A *majorization permutahedron* $M(v)$ is a polytope associated with a majorization $x \preceq v$ in $\Re^n$, defined by $M(v) = \{x \in \Re^n : x \preceq v\}$. By Rado's theorem ([2]) $M(v)$ is the convex hull of all permutations of v. Several properties of these polytopes are investigated and a connection to discrete convexity is established. These results are used to obtain a generalization of the Gale-Ryser theorem for $(0,1)$-matrices with given line sums.

[1] R.A. Brualdi, *Combinatorial Matrix Classes*, Encyclopedia of Mathematics, Cambridge University Press. 2006.
[2] A.W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.

Why are minimum rank of graph problems interesting? (In my opinion)
Shaun Fallat, University of Regina
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Mon 15:00, Room C

Given a graph $G = (V, E)$ on n vertices, we may associate a number of collections of matrices whose zero-nonzero pattern is constrained in some fashion by the edges of G. For example, let $S(G)$ be the set of all real symmetric matrices $A = [a_{ij}]$, such that if $i \neq j$, then $a_{ij} \neq 0$ iff $\{i, j\} \in E$. The parameter $mr(G)$ defined as the $\min\{rank(A) : A \in S(G)\}$ is known as the *minimum rank of* G (with respect to $S(G)$). Other notions of "minimum rank" may be defined in a similar manner. It is striking that for many different classes of graphs, notions of minimum rank are intimately connected with purely combinatorial graph parameters for that class. I intend to survey a number of results (new and old) along these lines to offer my perspective on why I think minimum rank parameters are interesting and worthwhile.

On the connection between weighted graphs and independence number
Miriam Farber, Technion - Israel Institute of Technology, Israel
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Tue 12:15, Room C

In this paper we generalize the concept of the Merris index of a graph by considering weighted Laplacians and obtain a better upper bound for the independence number, namely, the minimum, on all possible weights, of the Merris index. We refer to this bound as weighted Merris index and show that in many cases, for example, regular bipartite graphs, it is equal to the independence number. Complete graphs are an example of a strict inequality. In order to construct graphs for which equality holds we study what happens to such graphs when an edge or a vertex are added, and find sufficient conditions for equality for the new graphs.. We also give some insights, using the independence number, on the vertices that are contained in the maximal independence set.

[1] Bojan Mohar, "Graph Theory, Combinatorics, and Applications", Vol. 2, Ed. Y. Alavi, G. Chartrand, O. R. Oellermann, A. J. Schwenk, Wiley, 1991, pp. 871-898.
[2] Felix Goldberg and Gregory Shapiro, "The Merris index of a graph", Electronic Journal of Linear Algebra, vol. 10 (2003), pp. 212-222.
[3] Kinkar Ch. Das, R.B. Bapat, " A sharp upper bound on the largest Laplacian eigenvalue of weighted graphs", Linear Algebra and its Applications 409 (2005) 153-165
[4] Roger A.Horn and Charles R.Johnson, Matrix Analysis,1985 pp.181
[5] Russell Merris. Laplacian matrices of graphs: a survey. Linear Algebra Appl., 197/8:143-176, 1994.
[6] W.N. Anderson, T.D. Morley, "Eigenvalues of the Laplacian of a graph", Lin. Multilin. Algebra 18 (1985) 141-145.

Joint work with Abraham Berman (Technion - Israel Institute of Technology)

Graphs cospectral with Kneser graphs.
Willem Haemers, Tilburg University
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Mon 11:25, Room C

An important problem in spectral graph theory is to decide which graphs are determined by the spectrum. In this talk we consider the famous Kneser graphs $K(n, k)$. The main result is the construction of graphs cospectral but nonisomorphic to $K(n, k)$ when $n = 3k - 1$, $k > 2$ and for infinitely many other pairs (n, k). We also consider related graphs in the Johnson association scheme.

Joint work with Farzaneh Ramezani(IPM)

Average minimum rank of graphs of fixed order
Leslie Hogben, Iowa State University and American Institute of Mathematics
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Tue 11:25, Room C

The minimum rank of a simple graph G is defined to be the smallest possible rank over all real symmetric matrices whose ijth entry (for $i \neq j$) is nonzero whenever $\{i, j\}$ is an edge in G and is zero otherwise. The average minimum rank over all labeled graphs of order n is investigated by determining bounds for the expected value of minimum rank of $G(n, \frac{1}{2})$, the usual Erdős-Rényi random graph on n vertices with edge probability $\frac{1}{2}$.

Joint work with Tracy Hall (Brigham Young University), Ryan Martin (Iowa State University), Bryan Shader (University of Wyoming)

Eigenvalues, Multiplicities and Graphs: An Update
Charles R. Johnson, College of William and Mary
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Mon 15:25, Room C

We survey recent results about the possible lists of multiplicities occurring among the eigenvalues of a Hermitian matrix with a given graph. Some interesting problems will be mentioned.

The minimum rank of a graph containing a k-clique

Raphael Loewy, Technion-Israel Institute of Technology, Haifa, Israel
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Tue 11:00, Room C

Let G be an undirected graph on n vertices and let F be a field. We denote by $S(F,G)$ the set of all $n \times n$ symmetric matrices with entries in F and whose graph is G, and by $mr(F,G)$ the minimum rank of all matrices in $S(F,G)$. In this talk we consider $mr(F,G)$ when G contains a k-clique. It is known that if F is an infinite field then $mr(F,G) \le n-k+1$. The validity of this upper bound for $mr(F,G)$ when F is a finite field is discussed.

Cut-norms and spectra of matrices

Vladimir Nikiforov, University of Memphis, Memphis, Tennessee, USA
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Mon 12:15, Room C

In 1997, Frieze and Kannan introduced and studied the cut-norm $\|A\|_{\square}$ of an $m \times n$ matrix $A = [a_{ij}]$, defined by

$$\|A\|_{\square} = \max_{X \subset [m],\ Y \subset [n]} \frac{1}{mn} \left| \sum_{i \in X, j \in Y} a_{ij} \right|.$$

Ever since then this parameter kept getting new attention. This talk presents inequalities between two versions of the cut-norm and the two largest singular values of arbitrary complex matrices. These results extend, in particular, the well-known graph-theoretical Expander Mixing Lemma and give a hitherto unknown converse of it. Furthermore, they imply a solution of a problem of Lovász, and give a spectral sampling theorem, which informally states that almost all principal submatrices of a real symmetric matrix are spectrally similar to it.

Eigenvalues and ordered multiplicities for matrices associated with a graph

Carlos M. Saiago, Universidade Nova de Lisboa, Portugal
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Mon 15:50, Room C

For a given tree T let $\mathcal{S}(T)$ denote the set of all symmetric matrices whose graph is T. A question for a given tree T is the following Inverse Eigenvalue Problem: If T has n vertices, exactly which sets of n real numbers (including multiplicities) occur as the spectrum of A for some $A \in \mathcal{S}(T)$. Another problem for T is the characterization of the lists of multiplicities, ordered by numerical order of the underlying eigenvalues (ordered multiplicities), that occur among matrices in $\mathcal{S}(T)$. Though the solution for these two problems is known for certain classes of trees (see [1], [2], [3]), the problem is, in general, open. When T is either a generalized star or a double generalized star, such two problems are equivalent, i.e., the only constraint on existence of a matrix in $\mathcal{S}(T)$ with prescribed spectrum is the existence of the corresponding list of ordered multiplicities.

The following simple observation is the purpose of this presentation: Given the spectrum of an n-by-n symmetric matrix B whose graph is a tree, there is a double generalized star T and a matrix $A \in \mathcal{S}(T)$ with the same spectrum as B.

[1] C. R. Johnson and A. Leal-Duarte. On the possible multiplicities of the eigenvalues of an Hermitian matrix whose graph is a given tree. *Linear Algebra and Its Applications* 348:7–21 (2002).

[2] C. R. Johnson, A. Leal-Duarte and C. M. Saiago. Inverse eigenvalue problems and lists of multiplicities of eigenvalues for matrices whose graph is a tree: the case of generalized stars and double generalized stars. *Linear Algebra and Its Applications* 373:311–330 (2003).

[3] Francesco Barioli and Shaun Fallat. On the eigenvalues of generalized and double generalized stars. *Linear Multilinear Algebra* 53(4):269–291 (2005).

Integral, square-integral graphs and perfect state transfer

D. Stevanović, University of Primorska, Slovenia, University of Niš, Serbia and University of Novi Sad, Serbia
dragance106@yahoo.com
Mon 11:50, Room C

It has been shown earlier that the necessary condition for the existence of a perfect state transfer in a quantum spin network is that the whole adjacency spectrum of the underlying graph has the form $a_1\sqrt{b}, a_2\sqrt{b}, \ldots, a_n\sqrt{b}$, for some integers $a_1, \ldots, a_n$ and b. We will call such graphs the *square-integral* graphs. Note that for $b = 1$ we get the usual integral graphs.

In the talk we will survey the known results on graphs with perfect state transfer, and then determine which of the known 4-regular integral graphs and the semiregular bipartite square-integral graphs with small vertex degree have perfect state transfer. We will also determine the conditions which ensure that the perfect state transfer property is preserved under NEPS of graphs.

Linear Algebra Education

Avi Berman, Technion - Israel Institute of Technology, Haifa, Israel
Steve J. Leon, University of Massachusetts, USA

Principles and tools in teaching linear algebra

Avi Berman, Tachnion, Haifa, Israel

Fri 12:15, Room A

What Have I Learned?

Jane Day, Mathematics Department San Jose State University
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Fri 11:00, Room A

Linear algebra is my favorite subject to teach. I've always been interested in how people learn and in teaching styles that might help students better understand and appreciate the power of linear algebra. I've been impressed by insights from the Linear Algebra Curriculum Study Group, the MAA, Jean Piaget, Maria Montessori, and from Guershon Harel and other colleagues. I've learned from observing other people's classes. I've seen that students learn differently, that some but not all really benefit from geometric visualization, applications, computer use and/or group work, and that my (supposedly crystal clear) phrasing can be ambiguous to them. I've found that student attitudes can be very different at another college. I will discuss such experiences and some methods I've tried that seemed to help or not.

To be announced

Guershon Harel, University of California, San Diego, USA

Tue 16:45, Room A

The use of Classroom Response Systems (clickers) in teaching linear algebra: Still more questions than answers

Boris Koichu, Technion Israel Institute of Technology

Tue 17:10, Room A

Classroom Response Systems (clickers) are available and rapidly disseminating technology of enhancing interactions in large-size classes. This technology enables students to individually respond to various multiple-choice questions asked by the lecturer during the lesson. The talk will focus on the following questions: What are some of the strategies of using clickers in large-size linear algebra classes? What are the affordances and limitations? What are the effects of using clickers on the students' learning, teaching practices and pedagogical knowledge of the lecturers? These questions will be discussed based on the review of the growing literature on the subject and on the preliminary results of an on-going teaching experiment conducted in the context of two basic linear algebra courses at the Technion.

Joint work with Eman Atrash, Israel Institute of Technology

Contents of Linear Algebra with Sage and Mobile Sage environment

San-Gu Lee, Sungkyunkwan University, Korea
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Fri 11:50, Room A

From the experience gained by students in our Linear Algebra class, we wish that our students can attain a better understanding of mathematical concepts and further they can be equipped with a tool to deal with some real world problems. Sage is an open-source mathematics software system. It combines the power of many existing open-source packages into a common Python-based interface. We have tried to adopt Sage for our Linear Algebra class. It worked beautifully without much cost. We will try to share our contents and experience that came from teaching of Linear Algebra in Sage and Mobile Sage environment.

Joint work with Duk-Sun Kim

The second undergraduate level course in linear algebra

Steve Leon, University of Massachusetts Dartmouth, USA

Fri 15:00, Room A

In this talk we will review briefly the recommendations made twenty years ago by the NSF sponsored Linear Algebra Corriculum Study Group. We will discuss what topics should be covered in undergraduate linear algebra courses and give reasons why we believe a second course in linear algebra should be required for all mathematics majors. The speaker will outline a number of alternatives for possible second courses. He will describe one such course where students work together in teams on projects and apply linear algebra to problems in areas such as digital imaging, computer animation, and coordinate metrology. Some of these projects may involve original undergraduate level research.

Using an economics model for teaching linear algebra

Edgard Possani, Instituto Tecnológico Autónomo de México, México
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Tue 17:35, Room A

In this talk we will present an approach to teaching linear algebra using models. In particular, we are interested in designing problems that meet the models and modeling [1] approach, and in analyzing students learning process under an APOS [2] perspective. We will present a short illustration of the analysis of an economics problem related to production. This problem elicits the use of several linear algebra concepts related to vector space. Previous work has highlighted the importance of using realistic problems in the teaching of linear algebra. Here we will address the use of learning trajectories, which together with a genetic decomposition has allowed us to design specific teaching sequences to help students develop the constructions needed to learn the desired concepts. The abovementioned economics problem has already been used in the classroom by several researchers who also teach linear algebra. We will present an analysis of the learning trajectory, and describe the actual learning process, its outcomes together with students' difficulties and modeling strategies. In the process of solving the problem students need to analyze specific sets of data, and this analysis helps them discriminate which data sets are better suited for finding unique solutions for the problem, and which conditions are necessary for the selection of the appropriate data. We have found that this problem helps the students give meaning to more complex algebra concepts such as base, linear independence, generating set, among others related to vector space. The realistic setting of the problem motivates them to carry out a deeper kind of mathematical analysis. We believe this approach promotes students' significant development of mathematical reasoning in a meaningful and realistic setting.

[1] R. Lesh & H. Doerr. Beyond Constructivist: A Model & Modelling Perspective on Mathematics Teaching, Learning and Problems Solving, Laurence Erlabaum Associates NH, 2003.

[2] E. Dubinksy. Reective Abstraction in Advanced Mathematical Thinking, in Advanced Mathematical Thinking (D. Tall, ed.), Kluwer (1991), 95-126.

Joint work with M. Trigueros (ITAM), G. Preciado (ITAM), D. Lozano (ITAM)

Questions about Teaching, Teaching Mathematics and Teaching Linear Algebra

Frank Uhlig,

Fri 11:25, Room A

Questions are the engine of our understanding and food for the development of our consciousness. What is there? What should be there? What am I doing or trying to do? Why so? What are we doing? How can I achieve my goal(s)? How will we achieve our goals? What are the intended consequences, the unintended ones? Can my goal(s) be achieved? What are the costs and success? Can our goals be achieved, at what cost and success? Questions of What, Why, How? and their

negations What not, Why not, How not? all play a helpful role in assessing our individual and group efforts and the success of teaching the next and after next generation.
Question 1: Why are we still teaching math today? (Why not just calculator/computer literacy?)
Question 2: Why are we teaching linear algebra and matrix theory? (Why not just introduce MATLAB in freshman year?)
Question 3: Why are we teaching known unstable algorithms in college algebra and linear algebra courses? Why are our textbooks full of these? (Why not give our students correct and useful information?)
Question 4: Why is linear (in)dependence the students' and our stumbling block in linear algebra courses and books? (Why not progress to eigen structures of normal matrices, the Schur normal form, matrix factorizations and the SVD?)
Question 5: Why are US high school graduates lagging behind? (Why not teach concepts and exploration from Kindergarden on?)
Question 6: Who am I? For and by myself, and for my students?

Contributed Minisymposia

Application of Linear and Multilinear Algebra in Life Sciences and Engineering

Shmuel Friedland, University of Illinois, Chicago, USA,
Amir Niknejad, The College of Mount Saint Vincent, Riverdale, NY USA

This Mini Symposium will bring together Scientists who use Linear algebra and Multilinear Algebra in their respected fields. The focus is on problems arising in molecular biology, biomedicine and engineering. Most application is related to the processing of biological and chemical data, Drug Discovery, including biological sequences, gene expression data or gene networks, functional genomics, gene network reconstruction reconstruction and Neural Networks. The tools include but not limited to dimension reduction techniques such as Singular Value Decomposition (SVD), Generalized Singular Value Decomposition (GSVD),Principal component analysis (PCA), spectral clustering, Latent Semantic Indexing, Nonlinear Dimension reduction, Support Vector Machine(SVM). The mini symposium will address both deterministic and stochastic frameworks.

Spectral Theorems of Karlin for Evolutionary Dynamics

Lee Altenberg, University of Hawai'i at Manoa
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Tue 11:00, Room Galilei

The dynamics of Darwinian evolution result from the fertile interaction of 'dispersing' operators (mutation, recombination, and other transformations of heritable states) and a 'concentrating' operator (natural selection). Analysis of the dynamics typically arrives at matrices that are products of stochastic and non-negative diagonal matrices, and the spectral radii of such products are shown by Karlin (1982) to decrease under two different forms of 'more' dispersion. Originally developed to analyze genetic diversity in subdivided populations, Karlin's theorems and their extensions have applications to anti-viral therapy, quasispecies, the evolution of genetic systems, recombination and mutation rates (Altenberg and Feldman, 1987; Altenberg, 2009), coupled maps, and other areas, which are here described.

[1] Karlin, S., 1982. Classification of selection-migration structures and conditions for a protected polymorphism. Pages 61–204 in M. K. Hecht, B. Wallace, and G. T. Prance, eds. Evolutionary Biology, volume 14. Plenum.
[2] Altenberg, L. and Feldman, M. W. 1987. Selection, generalized transmission, and the evolution of modifier genes. I. The reduction principle. Genetics 117:559–572.
[3] Altenberg, L. 2009. The evolutionary reduction principle for linear variation in genetic transmission. Bulletin of Mathematical Biology 71:1264–1284.

Linear algebra issues in a fast algorithm for a large scale nonlinear nonlocal model of the inner ear

D. Bertaccini, Università di Roma "Tor Vergata", Roma. Italy
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Fri, 11:25, Room C

Recently, we proposed in [1] a fast second order package for a nonlinear, nonlocal model for the inner ear improving algorithms [2,3,4] for inner ear simulation of the evolution of the transverse displacement of the basilar membrane at each cochlear place. This information allows one to follow the forward and backward propagation of the traveling wave along the basilar membrane, and to evaluate the otoacoustic response from the time evolution of the stapes displacement.

In this talk, we illustrate the main results and performances of the numerical linear algebra core of the algorithms in [1] and [4] and, in particular, will focus on invertibility and conditioning of matrices, convergence of inner iterations, preconditioning and computational complexity issues.

[1] D. Bertaccini, R. Sisto, Fast numerical solution of a nonlinear nonlocal feed-forward cochlear model, submitted, 2010.
[2] D. Bertaccini, S. Fanelli, Computational and conditioning issues of a discrete model for cochlear sensorineural hypoacusia", Applied Numerical Mathematics, vol. 59, pp. 1989-2001, 2009.
[3] Elliott S.J., Ku E.M., Lineton B., "A state space model for cochlear mechanics", Journal of the Acoustical Society of America, vol. 122, No.5, pp. 2759-2771, 2007.
[4] A. Moleti, N. Paternoster, D. Bertaccini, R. Sisto, F. Sanjust, Otoacoustic emissions in time-domain solutions of nonlinear nonlocal cochlear models, Journal of Acoustical Society of America (JASA) vol. 126, pp. 2425-2436, 2009.

Joint work with A. Moleti (Università di Roma "Tor Vergata", Roma) and R. Sisto (ISPESL research center, Roma)

Enhanced line search for blind channel identification based on the Parafac decomposition of cumulant tensors

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Thu, 11:50, Room C

Consider a baseband communication system with discrete-time model

$$y(n) = x(n) + v(n), \quad x(n) = (h * s)(n) := \sum_{l=0}^{L} h(l)s(n-l),$$

where $s(n)$ is the sequence of transmitted symbols, $h(n)$ is the channel impulse response, $v(n)$ is additive noise, and $y(n)$ is the observed channel output.

The goal of blind identification is to estimate $h(n)$ from the observed system output $y(n)$, after which the input signal $s(n)$ can be recovered.

One class of blind identification algorithms is based on fitting higher-order cumulants. This yields the following multilinear algebra problem: decompose a given third-order tensor T that has certain symmetry properties into a sum of rank-1 terms (this is known as the PARAFAC decomposition).

Because of the symmetry properties of T (the factors in the PARAFAC decomposition have a Hankel structure) the common alternating least squares (ALS) algorithm is not applicable. Recently, a single-step least-squares (SSLS) algorithm has been proposed as an alternative. This algorithm preserves the symmetry properties but it does not necessarily converge monotonically. Moreover, the conditions that guarantee the convergence are not known.

It is known that ALS-based PARAFAC algorithms can be significantly improved by applying an enhanced line search (ELS) procedure. Namely, new ELS algorithms are less sensitive to local optima and have higher convergence speed.

We compute the PARAFAC decomposition of T combining the SSLS algorithm with ELS. Our method converges monotonically. It preserves the symmetry and the Hankel structure. We derive an explicit solution for the optimal real and complex step in the line search.

Joint work with L. De Lathauwer (K.U.Leuven: Campus Kortrijk and E.E. Dept. (ESAT), Belgium)

Phylogenic invariants and tensors of border rank 4 at most in $\mathbb{C}^{4\times4\times4}$

Shmuel Friedland, Department of Mathematics, Statistics and Computer Science University of Illinois at Chicago, Chicago, Illinois 60607-7045, USA
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Tue, 12:15, Room Galilei

We first discuss briefly the notion of algebraic statistics, phylogenic trees and their invariants. Then we consider the model in which one parent gives rise to new 3 species. This model is characterized as the variety $\mathcal{R}(4, \mathbb{C}^{4\times4\times4})$ of tensors in $\mathbb{C}^{4\times4\times4}$ of border rank 4 at most. In this talk we characterize this variety, and show that it is cut out by certain homogeneous polynomials of degrees $5, 9, 16$.

[1] E.S. Allman and J.A. Rhodes, Phylogenic ideals and varieties for general Markov model, *Advances in Appl. Math.*, 40 (2008) 127-148.
[2] S. Friedland, On tensors of border rank l in $\mathbb{C}^{m\times n\times l}$, arXiv:1003.1968.

Uses and behaviour of large sample covariances matrices in computational molecular biology with small sample sizes

David C. Hoyle, University of Manchester, UK
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Thu, 12:15, Room C

Sample covariance matrices play an important role in many algorithms used within bioinformatics and computational molecular biology - from dimensionality reduction algorithms such as Principal Components Analysis (PCA) used to visualize experimental data, to construction of gene regulatory association networks used to uncover the functional links between genes. However, the number of genes measured in modern post-genomic assays is typically very much greater than the sample size. This high-dimensional small sample-size scenario can severely limit the accuracy of sample covariance eigenvalues and eigenvectors used as estimators of their population counterparts, and gives rise to interesting phase transition phenomena in the behaviour of the eigenvalues and eigenvectors. In this talk we will give a brief introduction to some of the uses of sample covariance matrices within modern computational molecular biology and describe recent results from both the statistical physics and statistics research communities on large sample covariance matrices.

Multiarray signal processing: tensor decomposition meets compressed sensing

Lek-Heng Lim, University of California, Berkeley
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Thu, 11:25, Room C

We discuss how recently discovered techniques and tools from compressed sensing can be used in tensor decompositions, with a view towards modeling signals from multiple arrays of multiple sensors. We show that with appropriate bounds on coherence, one could always guarantee the existence and uniqueness of a best rank-r approximation of a tensor. In particular, we obtain a computationally feasible variant of Kruskal's uniqueness condition with coherence as a proxy for k-rank. We treat sparsest recovery and lowest-rank recovery problems in a uniform fashion by considering Schatten and nuclear norms of tensors of arbitrary order and dictionaries that comprise a continuum of uncountably many atoms.

Joint work with Pierre Comon (University of Nice)

TBA

A. Niknejad, College of Mount Saint Vincent, Riverdale, New York, USA
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Fri, 11:50, Room C

On the spectra of Fibonacci-like operators and modeling invasions by fungal pathogens

Ivan Slapnicar, Technical University Berlin, Germany, on leave from University of Split, Croatia
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Tue, 11:50, Room Galilei

The first part of talk deals with the spectra of the infinite dimensional generalized Fibonacci and Fibonacci-like operators in l^1. The operators are related to Fibonacci sequence. In the second part of the talk the Leslie matrix model for the invasion of potato late blight (oomycete Phytophthora infestans) is discussed. The spectral analysis from the first part of the talk yields a prediction of the maximum speed of the spread of invasion.

Best matrix approximation: the case of filtering with variable memory

A. Torokhti, University of South Australia, Australia
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Thu, 11:00, Room C

This paper concerns the best linear causal operator approximation of the identity mapping subject to a specified variable finite memory constraint. The problem is motivated by Wiener-like filtering subject to causality and memory constraints [1]. The filter is interpreted as a linear operator. The causality and memory restrictions require that the approximating operator takes the form of a lower stepped matrix A. To find the best such matrix, we propose a new technique based on a block-partition into an equivalent collection of smaller blocks, $\{L_0, K_1, L_1, \ldots, K_\ell, L_\ell\}$ where each L_r is a lower triangular block and each K_r is a rectangular block and where ℓ is known [2]. The sizes of the individual blocks are defined by the memory constraints. We show that the best approximation problem for the lower stepped matrix A can be replaced by an equivalent collection of ℓ independent best approximation problems in terms of the matrices $[L_0], [K_1, L_1], \ldots, [K_\ell, L_\ell]$. The solution to each individual problem is found and a representation of the overall solution and associated error is given.

[1] A. Torokhti, P. Howlett, *Computational Methods for Modelling of Nonlinear Systems,* Elsevier, 397 p., 2007.
[2] A. Torokhti and P. Howlett, Best approximation of identity mapping: the case of variable memory, *J. Approx. Theory,* 143, 1, pp. 111-123, 2006.

Joint work with P. Howlett (University of South Australia)

Homotopies to solve Multilinear Systems
Jan Verschelde, University of Chicago at Illinois, U.S.A.
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Tue, 11:25, Room Galilei

Many applications in mechanism design lead to structured polynomial systems. For systems where only isolated solutions matter, homotopies that exploit multihomogeneous structures are well developed since [3], see also [4]. For mechanisms that move, computing the corresponding algebraic curves with a optimal number of solution paths requires an adaption of the numerical representation for these curves. In the line of our work [1,2], we report on our new algorithms to solve multilinear systems more efficiently.

[1] Y. Guan and J. Verschelde. Parallel implementation of a subsystem-by-subsystem solver. In *The proceedings of the 22th High Performance Computing Symposium, Quebec City, 9-11 June 2008*, pages 117–123. IEEE Computer Society, 2008.
[2] Y. Guan and J. Verschelde. Sampling algebraic sets in local intrinsic coordinates. `arXiv:0912.2751`, submitted for publication.
[3] A.P. Morgan and A.J. Sommese. A homotopy for solving general polynomial systems that respects m-homogeneous structures. *Appl. Math. Comput.*, 24(2):101–113, 1987.
[4] A.J. Sommese and C.W. Wampler. *The Numerical solution of systems of polynomials arising in engineering and science.* World Scientific, 2005.

Joint work with Yun Guan (University of Illinois at Chicago)

PCCA+ and Spectral Clustering in Computational Drug Design
M. Weber, Zuse Institute Berlin (ZIB), Germany
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Fri, 11:00, Room C

Long-term molecular simulation of the interaction of drug-sized molecules with their target proteins produces large data sets of conformational states. In order to analyze this data set in terms of metastable subsets of the dynamical process (and their time-scales), spectral clustering methods gain a lot of importance in the last years. Especially, Robust Perron Cluster Analysis (PCCA+) turned out to be the only suitable algorithm for extrapolating the time-scale of the simulation to the time-scale of biological processes.

Nonlinear Eigenvalue Problems

Daniel Kressner, Institut fuer Mathematik, ETH, Zurich
Volker Mehrmann, Institut fuer Mathematik, TU Berlin

A variety of applications in science and engineering lead to eigenvalue problems that are nonlinear in the eigenvalue parameter. This includes polynomial, rational, as well as genuinely nonlinear eigenvalue problems. In recent years, tremendous progress has been made in addressing such eigenvalue problems, both on the theoretical and the computational side. The purpose of this minisymposium is to survey these developments and point out new directions in this area. A range of topics will be covered, including linearization, perturbation theory, structure preservation, numerical methods and emerging applications such as photonic band structure calculation.

Classification of Hermitian Matrix Polynomials with Real Eigenvalues of Definite Type
M. Al-Ammari, The University of Manchester, UK
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Fri 11:00, Room Fermi

The spectral properties of Hermitian matrix polynomials with real eigenvalues have been extensively studied, through classes such as the definite or definitizable pencils, definite, hyperbolic, or quasihyperbolic matrix polynomials, and overdamped or gyroscopically stabilized quadratics. We give a unified treatment of these and related classes that uses the eigenvalue type (or sign characteristic) as a common thread. Equivalent conditions are given for each class in a consistent format. We show that these classes form a hierarchy, all of which are contained in the new class of quasidefinite matrix polynomials. As well as collecting and unifying existing results, we propose a new characterization of hyperbolicity in terms of the distribution of the eigenvalue types on the real line. By analyzing their effect on eigenvalue type, we show that homogeneous rotations allow results for matrix polynomials with nonsingular or definite leading coefficient to be translated into results with no such requirement on the leading coefficient, which is important for treating definite and quasidefinite polynomials.

[1] M. Al-Ammari and F. Tisseur. Hermitian matrix polynomials with real eigenvalues of definite type- part I: Classification, MIMS Eprint 2010.9, Manchester Institute for Mathematical Sciences, The University of Manchester, UK, Jan. 2010. 24 pp.
[2] D. S. Mackey, N. Mackey, C. Mehl, and V. Mehrmann. Vector spaces of linearizations for matrix polynomials. SIAM

J. Matrix Anal. Appl., 28(4):971–1004, 2006.

Joint work with F. Tisseur (The University of Manchester)

Eigenvalue enclosures for the Dirac operator

L. Boulton, Heriot-Watt University, United Kingdom
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Fri 15:50, Room Fermi

The variational formulation of spectral problems associated to relativistic and non-relativistic atomic structures leads to rigorous upper bounds for the energy eigenvalues. This approach has proven to be highly valuable in the numerical estimation of these eigenvalues by finite-basis projection methods. Less effort has been devoted to the investigation of rigourous lower bounds for the spectrum of hamiltonians. This is due, in part, to the fact that most available techniques require a priori information, not usually at hand, about the problem being considered. Moreover, these procedures are often several orders of magnitude less accurate than their "upper-bound" counterparts. In this talk we will report on rigourous methods for computation of eigenvalue enclosures of Dirac operators. We will demonstrate the applicability of these techniques and will show how matrix polynomials and functions arise naturally in them. We will also report on outcomes of various numerical experiments performed on benchmark potentials.

Joint work with J. Dolbeault (Université Paris Dauphine)

Linearizations of rectangular matrix polynomials

Fernando De Terán, Universidad Carlos III de Madrid, Spain
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Thu 17:10, Room Fermi

Linearizations of regular matrix polynomials have been widely studied and they have shown to be a useful tool in several areas including the Polynomial Eigenvalue Problem. Also, linearizations of square singular matrix polynomials have been recently studied by the authors in a series of papers. By contrast, very little is known about linearizations of rectangular matrix polynomials. In this talk, we will present some results regarding general linearizations of rectangular matrix polynomials and we will also introduce a new family of strong linearizations of rectangular polynomials extending the family of *Fiedler pencils*. This family, which includes the *first* and *second companion forms*, was introduced in [1] for regular matrix polynomials and later extended in [2] to square singular polynomials. We will show that this family of linearizations enjoys several interesting properties that may be useful for future applications.

[1] E. N. Antoniou and S. Vologiannidis, A new family of companion forms of polynomial matrices, Electron. J. Linear Algebra, 11 (2004), pp. 78–87.
[2] F. De Terán, F. M. Dopico, and D. S. Mackey, Fiedler companion linearizations and the recovery of minimal indices, submitted to SIAM J. Matrix Anal. Appl.

Joint work with Froilán M. Dopico (Universidad Carlos III de Madrid) and D. Steven Mackey (Western Michigan University)

Generic spectral perturbation results for matrix polynomials

Froilán M. Dopico, Universidad Carlos III de Madrid, Leganés, Spain
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Thu 17:35, Room Fermi

In this talk, we deal with two spectral perturbation problems for matrix polynomials. In the first one, we consider the change of the elementary divisors of a regular matrix polynomial under a perturbation of low rank, while in the second one, we consider the first order perturbation term in the perturbation expansions of the eigenvalues of a square singular matrix polynomial. A common feature of these two problems is that, although the behavior of the considered magnitudes under an arbitrary perturbation may be very complicated, the "generic" behavior can be described in a compact and sharp way, where by "generic" behavior we understand the one that holds for all perturbations except those in a proper algebraic manifold of zero measure in the set of perturbations.

Joint work with Fernando De Terán (Universidad Carlos III de Madrid)

On nonlinear eigenvalue problems with applications to absorptive photonic crystals

C. Engström, ETH Zurich, Switzerland
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Fri 15:00, Room Fermi

Dielectric and metallic photonic crystals are promising materials for controlling and manipulating electromagnetic waves [1]. For frequency independent material models considerable mathematical progress has been made [2]. In the frequency dependent case, however, the nonlinearity of the spectral problem complicates the analysis. We study the spectrum of a scalar operator-valued function with periodic coefficients, which after application of the Floquet transform become a family of spectral problems on the torus. The frequency dependence of the material parameters lead to spectral analysis of a family of holomorphic operator-valued functions. We show that the spectrum for a passive material model consists of isolated eigenvalues of finite geometrical multiplicity. These eigenvalues depend continuously on the quasi momentum and all non-zero eigenvalues have a non-zero imaginary part whenever losses (absorption) occur [3].

Lorentz permittivity model, which is a common model for solid materials, lead to a rational eigenvalue problem. We study both the self-adjoint case and the non-self-adjoint case. Moreover, a high-order discontinuous Galerkin method is used to discretize the operator-valued function, and the resulting matrix problem is transformed into a linear eigenvalue problem. Finally, we use an implicitly restarted Arnoldi method to compute approximate eigenpairs of the sparse matrix problem.

[1] K. Sakoda, *Optical properties of photonic crystals*, Springer-Verlag, Heidelberg, 2001.
[2] P. Kuchment, *Floquet theory for partial differential equations*, Birkhäuser, Basel, 1993.
[3] C. Engström, *On the spectrum of a holomorphic operator-valued function with applications to absorptive photonic crystals*, To appear.

Computation and continuation of invariant pairs for polynomial and nonlinear eigenvalue problems

D. Kressner, ETH Zurich, Switzerland
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Fri 15:25, Room Fermi

We consider matrix eigenvalue problems that are polynomial or genuinely nonlinear in the eigenvalue parameter. One of the most fundamental differences to the linear case is that distinct eigenvalues may have linearly dependent eigenvectors or even share the same eigenvector. This can be a severe hindrance in the development of general numerical schemes for computing several eigenvalues of a polynomial or nonlinear eigenvalue problem, either simultaneously or subsequently. The purpose of this talk is to show that the concept of invariant pairs offers a way of representing eigenvalues and eigenvectors that is insensitive to this phenomenon. We will demonstrate the use of this concept with a number of numerical examples and discuss continuation methods for invariant pairs.

[1] T. Betcke and D. Kressner. Perturbation, Computation and Refinement of Invariant Pairs for Matrix Polynomials. Technical report 2009-21, Seminar for applied mathematics, ETH Zurich, July 2009. Revised February 2010.
[2] D. Kressner. A block Newton method for nonlinear eigenvalue problems. Numer. Math., 114(2):355–372, 2009.

Joint work with Timo Betcke (University of Reading)

Leave it to Smith: Canonical Forms for Structured Matrix Polynomials, Part II

Niloufer Mackey, Western Michigan University, Kalamazoo, USA
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Thu 15:25, Room Fermi

Polynomial eigenvalue problems arise in many applications, and often the underlying matrix polynomial P is structured in some way. A much used computational approach to such problems starts with a linearization such as the companion form of P, and then applies a general purpose algorithm to the linearization. But when P is structured, it can be advantageous to use a linearization with the same structure as P, if one can be found. It turns out that there are structured polynomials for which a linearization with the same structure does not exist. Using the Smith form as the central tool, we describe which matrix polynomials from the classes of alternating, palindromic, and skew-symmetric polynomials allow a linearization with the same structure.

[1] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, Jordan Structures of Alternating Matrix Polynomials, Linear Alg. Appl., v. 432:4, pp. 867–891, 2010.
[2] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, Smith Forms of Palindromic Matrix Polynomials, In preparation.

Joint work with D. Steven Mackey (Western Michigan University), Christian Mehl (Technische Universität Berlin), Volker Mehrmann (Technische Universität Berlin).

Spectral Equivalence and the Rank Theorem for Matrix Polynomials

D. Steven Mackey, Western Michigan University, Kalamazoo, MI, USA
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Thu 16:45, Room Fermi

We investigate the extent to which two matrix polynomials of different sizes and degrees, regular or singular, square or rectangular, can have the same (scalar) spectral data, i.e., the same finite and infinite elementary divisors. The classical example of this phenomenon is the well-known concept of strong linearization. Taking this example as a prototype, we introduce the notion of spectral equivalence, describe its basic properties, and give a variety of examples. For matrix polynomials $P(\lambda)$ that are singular, the minimal indices of $P(\lambda)$ are another type of scalar spectral-like data that encode important properties of the left and right nullspaces of $P(\lambda)$. When two singular matrix polynomials are spectrally equivalent, what are the possible relationships between their minimal indices? For example, can they be equal? In trying to answer these questions, we prove the Rank Theorem for Matrix Polynomials, a simple but fundamental relation between elementary divisors, minimal indices, and rank that holds for any matrix polynomial.

Earlier results analyzing the relationship between the minimal indices of a singular polynomial and those of several particular classes of strong linearization can be found in the recent papers [1] and [2].

[1] F. De Terán, F.M. Dopico, and D.S. Mackey, Linearizations of singular matrix polynomials and the recovery of minimal indices, Electron. J. Linear Alg., 18 (2009), pp. 371–402.
[2] F. De Terán, F.M. Dopico, and D.S. Mackey, Fiedler companion linearizations and the recovery of minimal indices, submitted to SIAM J. Matrix Anal. Appl.

Joint work with Fernando De Terán (Universidad Carlos III de Madrid) and Froilán M. Dopico (Universidad Carlos III de Madrid).

Leave it to Smith: Canonical Forms for Structured Matrix Polynomials, Part I

Christian Mehl, Technische Universität Berlin, Germany
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Thu 15:00, Room Fermi

Polynomial eigenvalue problems arise in many applications, and often the underlying matrix polynomial P is structured in some way. A much used computational approach to such problems starts with a linearization such as the companion form of P, and then applies a general purpose algorithm to the linearization. But when P is structured, it can be advantageous to use a linearization with the same structure as P, if one can be found. It turns out that there are structured polynomials for which a linearization with the same structure does not exist. Using the Smith form as the central tool, we describe which matrix polynomials from the classes of alternating, palindromic, and skew-symmetric polynomials allow a linearization with the same structure.

[1] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, Jordan Structures of Alternating Matrix Polynomials, Linear Alg. Appl., v. 432:4, pp. 867–891, 2010.
[2] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann, Smith Forms of Palindromic Matrix Polynomials, In preparation.

Joint work with D. Steven Mackey (Western Michigan University), Niloufer Mackey (Western Michigan University), Volker Mehrmann (Technische Universität Berlin).

Nonlinear eigenvalue problems in acoustic field computation

V. Mehrmann, TU Berlin, Germany
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Fri 12:15, Room Fermi

We will discuss the numerical solution of large scale parametric eigenvalue problems arising in acoustic field problems. In current industrial applications a few eigenvalues in a specified region of the complex plane have be computed for nonlinear eigenvalue problems with several million degrees of freedom within an optimization loop. Based on geometric, topological or material changes the acoustic field within modern cars is then optimized on the basis of the eigenvalue computations.

We will discuss the currently used methods and their properties from a numerical and computational point of view. In particular we discuss homotopy methods and eigenvalue continuation techniques.

Joint work with T. Baumgarten (TU Berlin), C. Schröder (TU Berlin)

A "shift-and-deflate" technique for matrix polynomials

B. Meini, University of Pisa, Italy
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Fri 11:25, Room Fermi

Let $P(z) = \sum_{i=0}^{n} A_i z^i$ be a regular $k \times k$ matrix polynomial of degree n, and let $\lambda \in \mathbb{C}$, $u \in \mathbb{C}^k$, $u \neq 0$ such that $P(\lambda)u = 0$. Given $\mu \in \mathbb{C} \cup \{\infty\}$, the shift technique introduced in [2] allows to trasform the matrix polynomial $P(z)$ into a new $k \times k$ matrix polynomial $\tilde{P}(z)$ of degree n such that $\tilde{P}(\mu)u = 0$. That is, the root λ of $P(z)$ is *shifted* to the root μ of $\tilde{P}(z)$, and the remaining roots are kept unchanged. In [1] the authors show how to deflate a couple of known roots of a quadratic matrix polynomial $P(z)$, by trasforming $P(z)$ into a $(k-1) \times (k-1)$ matrix polynomial $\hat{P}(z)$, having as roots the unknown roots of $P(z)$. The aim of this talk is to show how the shift technique can be used to the same purpose in a much simpler way. Moreover, if $P(z)$ has a specific structure, like symmetric matrix coefficients, or palindromic structure, then the matrix polynomial $\hat{P}(z)$ can be constructed with the same structure of the original polynomial.

[1] D. Garvey, C. Munro, and F. Tisseur. Deflating Quadratic Matrix Polynomials with Structure Preserving Transformations. MIMS EPrint 2009.22, March 2009.

[2] C. He, B. Meini, and N. H. Rhee. A shifted cyclic reduction algorithm for quasi-birth-death problems. *SIAM J. Matrix Anal. Appl.*, 23(3):673–691, 2001/02.

Structure Preserving Transformations for Quadratic Matrix Polynomials

F. Tisseur, The University of Manchester, UK.
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Thu 15:50, Room Fermi

A structure preserving transformations (SPT) is a map transforming a quadratic matrix polynomial $Q(\lambda) = \lambda^2 A_2 + \lambda A_1 + A_0$ into a new quadratic $\widetilde{Q}(\lambda) = \lambda^2 \widetilde{A}_2 + \lambda \widetilde{A}_1 + \widetilde{A}_0$ isospectral to $Q(\lambda)$ (i.e., Q and $\widetilde{Q}$ have the same Jordan canonical form). Two essential points are

1. An SPT does not act on the polynomial Q: it is defined as an action on a linearization L of Q.
2. Computationally, an SPT can be applied by working only with the $n \times n$ coefficient matrices of Q, avoiding computations on the larger pencil L.

In this talk we describe the concept of SPTs and present recent developments involving them. SPTs are a novel and promising approach to solving quadratic eigenproblems.

Nonlinear low rank modification of a symmetric eigenvalue problem

H. Voss, Hamburg University of Technology, Germany
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Fri 11:50, Room Fermi

In a recent report Huang, Bai and Su [1] studied existence and uniqueness results and interlacing properties of nonlinear rank-one modifications of symmetric eigenvalue problem. In this talk we generalize the uniqueness conditions and we discuss generalizations to low rank modifications. Based on approximation properties of the Rayleigh functional we design numerical methods the local convergence of which are quadratic or even cubic. Numerical examples demonstrate their efficiency. We further consider low rank modifications of hyperbolic quadratic eigenvalue problems and more general nonlinear eigenvalue problems allowing for a minmax characterization of their eigenvalues.

[1] X. Huang, Z. Bai, Y. Su, Nonlinear rank-one modification of a symmetric eigenvalue problem, Technical Report, UC Davis, 2009, to appear in Math. Comp.

[2] H. Voss, K. Yildiztekin, Nonlinear low rank modification of a symmetric eigenvalue problem, Technical Report, Hamburg University of Technology, 2009, Submitted to SIAm J. Matrix Anal. Appl.

Joint work with K. Yildiztekin (Hamburg University of Technology)

Spectral graph theory

Vladimir S. Nikiforov, Department of Mathematical Sciences, The University of Memphis, TN,
Dragan Stevanovic, Faculty of Science and Mathematics, University of Nis, Serbia

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Spectral graph theory is a fast developing field in modern discrete mathematics with important applications in computer science, chemistry and operational research. By merging combinatorial techniques with algebraic and analytical methods it creates new approaches to hard discrete problems and gives new insights in classical Linear Algebra. The proposed minisymposium will bring together leading researchers on graph spectra to present their recent results and to discuss new achievements and problems. This meeting will further increase collaboration and boost the development of the field.

The structure of graphs with small M-indices

F. Belardo, University of Messina, Italy
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Tue 17:10, Room C

In this talk we consider simple graphs and as graph matrices the adjacency matrix $A(G)$, the Laplacian matrix $L(G) = D(G) - A(G)$ and the signless Laplacian matrix $Q(G) = D(G) + A(G)$. Let M-index be the largest eigenvalue of G with respect to the graph matrix M.

By synthesizing the results of [1,2,3], we show that almost all graphs whose A-index does not exceed $\sqrt{2+\sqrt{5}}$ are graphs whose $\{L, Q\}$-index does not exceed $2+\epsilon$, where $2+\epsilon \approx 4.38298$ is the real root of $x^3 - 6x^2 + 8x - 4$. Furthermore we consider the analogy between the structure of graphs whose A-index does not exceed $\frac{3}{2}\sqrt{2}$ and the structure of graphs whose $\{L, Q\}$-index does not exceed 4.5.

Finally, we discuss the analogies between $\sqrt{2+\sqrt{5}}$ (or $\frac{3}{2}\sqrt{2}$) as limit point for the index in the A-theory and $2+\epsilon$ (resp. 4.5) as limit point for the index in the $\{L, Q\}$-theory of graph spectra.

[1] F. Belardo, E.M. Li Marzi, S.K. Simić, Ordering graphs with index in the interval $(2, \sqrt{2+\sqrt{5}})$, Discrete Applied Math., 156/10 (2008), pp. 1670–1682.
[2] J.F. Wang, Q.X. Huang, F. Belardo, E.M. Li Marzi, On graphs whose signless Laplacian index does not exceed 4.5, Linear Algebra Appl., 431 issues 1–2 (2009), pp. 162–178.
[3] J.F. Wang, F. Belardo, Q.X. Huang, E.M. Li Marzi, On graphs whose Laplacian index does not exceed 4.5, submitted.

Graphs of given order and size and minimal algebraic connectivity

T. Bıyıkoğlu, Işık University, İstanbul, Turkey
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Wed 11:00, Room C

We investigate the structure of connected graphs that have minimal algebraic connectivity among all graphs with given number of vertices and edges. It has been conjectured [1] that such graphs are so called path-complete graphs. In this talk we show that the concept of *geometric nodal domains* can be used to derive some necessary conditions on the structure of graphs which have minimal algebraic connectivity. In particular we show that such extremal graphs consists of a chain of complete graphs which cannot have to many big cliques.

[1] S. Belhaiza et al., Variable neighborhood search for extremal graphs. XI. Bounds on Algebraic Connectivity, pp.1–16. In: D. Avis et al., Graph Theory and Combinatorial Optimization, New York, 2005. DOI: 10.1007/0-387-25592-3_1

Joint work with J. Leydold (WU Vienna, Austria)

Graph Eigenvalues in Combinatorial Optimization

Domingos M. Cardoso, Departamento de Matemática, Universidade de Aveiro, 3810-193 Aveiro, Portugal
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Tue 17:35, Room C

A number of remarkable spectral properties of graphs with applications in combinatorial optimization are surveyed and a few additional ones are introduced. Namely, several spectral bounds on the clique number, stability number, and chromatic number of graphs are analyzed and spectral graph tools for deciding about the existence of particular combinatorial structures (as it is the case of dominating induced matchings) are presented.

Decompositions of complete hypergraphs

S.M. Cioabă, University of Delaware, USA
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Tue 15:50, Room C

A classical result of Graham and Pollak states that the minimum number of complete bipartite subgraphs that partition the edges of a complete graph on n vertices is n-1. In this talk, I will describe our attempts at proving a natural hypergraph version of Graham-Pollak's theorem.

[1] S. M. Cioabă, A. Kündgen and J. Verstraëte, On decompositions of complete hypergraphs, *J. Combin. Theory, Series A* Volume 116, Issue 7, October 2009, Pages 1232-1234.

Joint work with Andre Kündgen (Cal State San Marcos) and Jacques Verstraëte(UC San Diego).

Some topics on integral graphs

D. Cvetković, Mathematical institute SANU, Belgrade, Serbia
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Tue 15:00, Room C

The M-spectrum of a graph is the spectrum of a graph matrix M (adjacency matrix A, Laplacian L, signless Laplacian Q, etc.). A graph is called *M-integral* if its M-spectrum consists entirely of integers. If the matrix M is fixed, we say, for short, *integral* instead of M-integral. A graph which is A-, L- and Q-integral is called *ALQ-integral*. A survey on integral graphs can be found in [1].

Integral graphs have recently found some applications in quantum computing, multiprocessor systems and chemistry.

Let G be a graph with the largest A-eigenvalue λ_1 and the diameter D. The quantity $(D+1)\lambda_1$ is called the *tightness* of G and is denoted by $t(G)$. There are exactly 69 non-trivial connected graphs G with $t(G) \leq 9$ and among them 14 graphs are A-integral [2]. We present a classification of A-integral graphs G with $t(G) < 24$.

In integral graphs on n vertices there exist sets of n independent integral eigenvectors. Such sets can be constructed using star partitions of graphs and can be useful in treating the load balancing problems in multiprocessor systems and some problems in combinatorial optimization.

Some results on L- and ALQ-integral graphs are presented as well.

[1] K. Balińska, D. Cvetković, Z. Radosavljević, S. Simić, D. Stevanović, A survey on integral graphs, Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat., **13**(2002), 42-65.
[2] D. Cvetković, T. Davidović, Multiprocessor interconnection networks with small tightness, Internat. J. Foundations Computer Sci., 20(2009), No. 5, 941-963.

Constructing infinite families of ALQ-integral graphs

Nair M.M. de Abreu, Federal University of Rio de Janeiro, Brazil
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Tue 15:25, Room C

Let $G = (V, E)$ be a simple graph on n vertices and $D = diag(d_1, \ldots, d_n)$ be the diagonal matrix of its vertex degrees. Let A be the *adjacency*, $L = A - D$ the *Laplacian* and $Q = A + D$ the *signless Laplacian* matrices of G. Since 1974, when Harary and Schwenk posed the question *Which graphs have integral spectra?* [1], the search for graphs whose adjacency eigenvalues or Laplacian eigenvalues are all integers (here called *A-integral graphs* and *L-integral graphs*, respectively) has been on. More recently, *Q-integral graphs* (graphs whose signless Laplacian spectrum consists entirely of integers) were introduced in the literature [2–6]. It is known that these three concepts coincide for regular graphs. Also, for bipartite graphs, L-integral and Q-integral graphs are the same. A graph is called ALQ-integral graph if it is simultaneously an $A-$, $L-$ and $Q-$integral graph. Among all 172 connected Q-integral graphs up to 10 vertices, there are 42 ALQ-integral graphs, but only one of them is neither regular and nor bipartite [4]. Our aim is to show how to construct infinite families of non regular and non bipartite graphs but all of them ALQ-integral graphs.

[1] F. Harary, A.J. Schwenk, *Which graphs have integral spectra?*, in: R. Bari, F. Harary (Eds.), "Graphs and Combinatorics", Lecture Notes in Mathematics, vol. 406, Springer, Berlin, 1974, pp. 45-51.
[2] D. Cvetković, P. Rowlinson, S. Simić, *Signless Laplacian of finite graphs*, Linear Algebra and its Applications 423 (2007) 155-171.

[3] S. Simić, Z. Stanić, *Q-integral graphs with edge-degrees at most five*, Discrete Math. 308 (2008) 4625-4634.
[4] Z. Stanić, *There are exactly 172 connected Q-integral graphs up to 10 vertices*, Novi Sad J. Math. 37 n. 2 (2007) 193-205.
[5] Z. Stanić, *Some results on Q-integral graphs*, Ars Combinatoria 90 (2009), 321-335.
[6] M.A.A. de Freitas *et al.*, *Infinite families of Q-integral graphs,* Linear Algebra Appl. (2009), doi:10.1016/j.laa.2009.06.029

Joint work with M.A.A. de Freitas (Federal University of Rio de Janeiro), R.R. Del-Vecchio (Fluminense Federal University) and C.T.M. Vinagre (Fluminense Federal University)

Distance spectral radius of trees

Aleksandar Ilić, Faculty of Sciences and Mathematics, University of Niš, Serbia
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Wed 11:50, Room C

Distance energy is a newly introduced molecular graph-based analog of the total π-electron energy, and it is defined as the sum of the absolute eigenvalues of the distance matrix. For trees and unicyclic graphs, the distance energy is equal to the doubled value of the distance spectral radius (the largest eigenvalue of the distance matrix).

We introduce two general transformations that strictly increase and decrease the distance spectral radius and provide an alternative proof that the path P_n has maximal distance spectral radius, while the star S_n has minimal distance spectral radius among trees on n vertices. We prove that a caterpillar $C_{n,d}$, obtained from the path P_d with all pendant vertices attached at the center vertex of P_d, has minimal spectral radius among trees with n vertices and diameter d. In addition, we characterize n-vertex trees with given matching number m which minimize the distance spectral radius. The extremal tree $A(n,m)$ is a spur, obtained from the star S_{n-m+1} by attaching a pendant edge to each of certain $m-1$ non-central vertices of S_{n-m+1}.

In conclusion, we pose some conjectures concerning the extremal trees with minimum or maximum distance spectral radius based on the computer search among trees on $n \leq 24$ vertices.

Algebraic connectivity and vertex-deleted subgraphs

Steve Kirkland, Hamilton Institute, National University of Ireland, Maynooth
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Tue 16:45, Room C

Given an undirected graph G, its Laplacian matrix L can be written as $L = D - A$, where A is the $(0,1)$ adjacency matrix for G, and D is the diagonal matrix of vertex degrees. The second smallest eigenvalue of L is known as the algebraic connectivity of G, and this quantity has been the subject of a good deal of work over the last several decades. In this talk, we will discuss some recent work relating the algebraic connectivity of a graph G to that of the graph formed from G by deleting a vertex and its incident edges.

Geometric nodal domains and extremal graphs with minimal k-th laplacian eigenvalue

J. Leydold, WU Vienna, Austria
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Wed 12:40, Room C

A method for characterizing graphs that have smallest (or largest) Laplacian eigenvalue within a particular class of graphs works as following: Take a Perron vector, rearrange the edges of the graph and compare the respective Rayleigh quotients. By the Rayleigh-Ritz Theorem we can draw some conclusions about the change of the smallest eigenvalue. This approach, however, does not work for the k-th Laplacian eigenvalue, as now we have to use the Courant-Fisher Theorem that involves minimization of the Rayleigh quotients with respect to constraints that are hard to control. In this talk we show that sometimes we can get local properties of extremal graphs by means of the concept of *geometric nodal domains* and *Dirichlet matrices.* This is in particular the case for the algebraic connectivity.

Joint work with T. Bıyıkoğlu (Işık University, İstanbul)

A generalization of Fiedler's lemma and some applications

Enide Andrade Martins, Departamento de Matemática, Universidade de Aveiro, Aveiro, Portugal
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Wed 11:25, Room C

In a previous paper, [1], a Fiedler's lemma introduced in [2] was used to obtain eigenspaces of graphs, and applied to graph energy. In this talk, this Fiedler's lemma is generalized and its generalization is applied to the determination of eigenvalues of graphs belonging to a particular family and also to the determinations of the graph energy (including lower and upper bounds).

[1] M. Robbiano, E. A. Martins and I. Gutman, Extending a theorem by Fiedler and applications to graph energy, MATCH Commun. Math. Comput. Chem. 64 (2010), 145-156.
[2] M. Fiedler, Eigenvalues of nonnegative symmetric matrices, Linear Algebra Appl. 9 (1974), 119-142.

Joint work with D.M. Cardoso (University of Aveiro), I. Gutman (University of Kragujevac) and Maria Robbiano (North Catholic University, Chile)

Forbidden subgraphs for some classes of treelike reflexive graphs

B. Mihailović, School of Electrical Engineering, University of Belgrade, Serbia
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Wed 12:15, Room C

Reflexive graphs are simple graphs whose second largest eigenvalue of $(0,1)$ - adjacency matrix does not exceed 2. Treelike graph, or a cactus, is a graph in which any two cycles are edge disjoint. Several classes of treelike reflexive graphs have been characterized through sets of maximal graphs. This paper presents another possible approach to the characterization of such graphs, i.e. via corresponding sets of forbidden subgraphs, and gives such sets for some classes of treelike reflexive graphs.

[1] V. Brankov, D. Cvetković, S. Simić, D. Stevanović: Simultaneous editing and multilabelling of graphs in system newGRAPH, Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. 17, pp. 112-121, 2006.
[2] D. M. Cvetković, M. Doob, H. Sachs: Spectra of Graphs-Theory and Application. Deutscher Verlag der Wissenschaften-Academic Press, Berlin-New York, 1980; second edition 1982; third edition, Johann Ambrosius Barth

Verlag, Heidelberg-Leipzig, 1995.
[3] D. Cvetković, L. Kraus, S. Simić: Discussing graph theory with a computer, Implementation of algorithms. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. Fiz., No. 716-No. 734, pp. 100-104, 1981.
[4] B. Mihailović, Z. Radosavljević: On a class of tricyclic reflexive cactuses. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat., 16, pp. 55-63, 2005.
[5] A. Neumaier, J. J. Seidel: Discrete hyperbolic geometry. Combinatorica, 3, pp. 219-237, 1983.
[6] Z. Radosavljević, M. Rašajski: Multicyclic treelike reflexive graphs. Discrete Math, Vol. 296/1, pp. 43-57, 2005.
[7] Z. Radosavljević, S. Simić: Which bicyclic graphs are reflexive? Univ. Beograd, Publ. Elektroteh. Fak., Ser. Mat., 7, pp. 90-104, 1996.

Joint work with Z. Radosavljević, M. Rašajski (School of Electrical Engineering, University of Belgrade, Serbia)

Matrix Inequalities — In Memory of Ky Fan

Chi-Kwong Li, College of William and Mary, Williamsburg, Virginia, USA
Fuzhen Zhang, Nova Southeastern University, Florida, USA

The purpose of this symposium is to stimulate researches in the area of matrix and operator inequalities and to provide an opportunity for mathematicians in the field to exchange ideas and share most recent developments and information.

On distance measures between positive semidefinite matrices and their applications in quantum information theory

Koenraad M.R. Audenaert, Royal Holloway, University of London, UK
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Mon 11:00, Auditorium

The theory of matrix inequalities has many applications in quantum information theory. In this talk we consider a number of distance measures between quantum states that are commonly used. Many problems in quantum information theory depend on finding useful relationships between these distance measures. These relationships can be generalised to matrix inequalities for positive semidefinite matrices. We give an overview of these inequalities, some of which are straightforward consequences of known inequalities, while others have been open problems for a long time, e.g. an inequality concerning the generalisation of the Chernoff distance between two probability distributions to quantum states. In addition, a number of conjectured inequalities are presented, too. In particular, we present some recent progress on a conjectured inequality involving N positive semidefinite matrices that would fill the only remaining gap in an argument extending the quantum Chernoff distance to N quantum states.

Joint work with Milan Mosonyi (Budapest University of Technology and Economics)

Matrix subadditivity inequalities

Jean-Christophe Bourin, Université de Franche-Comté, France
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Mon 11:25, Auditorium

This talk surveys several recent results of functional analytic spirit in matrix analysis. Most of these results are subadditivity inequalities for symmetric norms (or unitarily invariant norms) and concave functions of operators. In case of normal operators, this leads to some estimates for partitioned matrices.

$-(\sigma_1 - \sigma_2)^4 \geq 0$

Roger A. Horn, University of Utah, Salt Lake City, Utah USA
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Mon 11:50, Auditorium

In 1961, D. C. Youla discovered a block upper triangular form to which any square complex matrix A can be reduced by a unitary congruence, that is, $A \to UAU^T$, in which U is unitary. We revisit Youla's form and describe a canonical form for its diagonal blocks. The inequality in the title plays a key role in identifying the diagonal blocks associated with real negative eigenvalues of $A\bar{A}$.

Jensen matrix inequalities and direct sums

Fuad Kittaneh, University of Jordan, Jordan
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Mon 12:15, Auditorium

Let A, X and Y be n-by-n complex matrices such that A is positive semi-definite and X, Y are contractions. We prove that if f is an increasing convex function on $[0, \infty)$ such that $f(0) \leq 0$, then the eigenvalues of $f(|X^*AY|)$ are dominated by those of $X^*f(A)X \oplus Y^*f(A)Y$. Several related results are considered.

Joint work with J.-C. Bourin (Université de Franche-Comté) and O. Hirzallah (Hashemite University)

Operator Radii and Unitary Operators

Chi-Kwong Li, Department of Mathematics, College of William and Mary, Williamsburg, USA
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Tue 11:00, Auditorium

Let $\rho \geq 1$ and $w_\rho(A)$ be the operator radius of a linear operator A. Suppose m is a positive integer. It is shown that for a given invertible linear operator A acting on a Hilbert space, one has $w_\rho(A^{-m}) \geq w_\rho(A)^{-m}$. The equality holds if and only if A is a multiple of a unitary operator.

Joint work with Tsuyoshi Ando, Professor Emeritus, Hokkaido University.

Perturbation of Partitioned Hermitian Generalized Eigenvalue Problem

Ren-Cang Li, University of Texas at Arlington, TX, USA
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Tue 11:25, Auditorium

We are concerned with Hermitian positive definite generalized eigenvalue problem $A - \lambda B$ for partitioned

$$A = \begin{pmatrix} A_{11} & \\ & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & \\ & B_{22} \end{pmatrix},$$

where both A and B are Hermitian and B is positive definite. Bounds on how its eigenvalues varies when A and B are perturbed by Hermitian matrices. These bounds are generally of linear order with respect to the perturbations in the diagonal blocks and of quadratic order with respect to the perturbations in the off-diagonal blocks. The results for the special case of no perturbations in the diagonal blocks can be used to bound the changes of eigenvalues of a Hermitian positive definite generalized eigenvalue problem after its off-diagonal blocks are dropped, a situation occurs frequently in eigenvalue computations.

Stewart and Sun (1990) observed that different copies of a multiple eigenvalue for the generalized eigenvalue problem may behave very differently. Recently, Nakatsukasa (2009) successfully obtained quantitative estimates to explain the behavior. In this talk, we will present different estimates.

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One horse racing story, two card games, and three matrix theorems

YIU-TUNG POON, Iowa State University, U. S. A.
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Tue 11:50, Auditorium

Motivated by a horse racing story in ancient China, we consider two card games. Let D_1 and D_2 be two diagonal matrices whose diagonal entries correspond to the card values of the two players of the card games. There is a correspondence between the outcomes of the first card game and the possible inertia of a matrix of the form $P^tD_1P - Q^tD_2Q$, where P and Q are permutation matrices. It turns out that there is also a correspondence between the outcomes of the second card game and the possible inertia of a matrix of the form $U^*D_1U - V^*D_2V$, where U and V are complex unitary (or real orthogonal) matrices. Using the simple strategy in the ancient story, we describe all the possible outcomes of the card games and the inertia of the corresponding matrices $P^tD_1P - Q^tD_2Q$ and $U^*D_1U - V^*D_2V$. Related problems and results are also mentioned.

Joint work with Chi-Kwong Li (Department of Mathematics, The College of William and Mary, U.S.A.)

Loewner matrices and matrix convexity

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Tue 12:15, Auditorium

In this talk, our results on Loewner matrices $\left[\frac{f(p_i)-f(p_j)}{p_i-p_j}\right]$ in [1] and [2] will be presented. Moreover, related results are to be reported.

[1] R. Bhatia, T. Sano, Loewner matrices and operator convexity, Math. Ann. 344 (2009), no. 3, 703–716.
[2] R. Bhatia, T. Sano, Positivity and conditional positivity of Loewner matrices, to appear in Positivity.

Inequalities in Construction of Higher Rank Numerical Ranges

RAYMOND NUNG-SING SZE, Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Hong Kong
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Thu 11:00, Auditorium

Given a positive integer k, the higher rank numerical range $\Lambda_k(A)$ of a matrix A is the set of complex λ such that for some rank k projection P we have $PAP = \lambda P$. It has been shown that the set $\Lambda_k(A)$ can be constructed by infinitely many inequalities of A. In particular, only finite many inequalities are needed if A is normal. In this talk, we revisit and demonstrate these constructions and related results.

Joint work with H.L. Gau (National Central University), C.K. Li (College of William and Mary), and Y.T. Poon (Iowa State University)

Determinant and Pfaffian of sum of skew symmetric matrices

TIN-YAU TAM, Auburn University, USA
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Thu 11:25, Auditorium

We completely describe the determinants of the sum of orbits of two real skew symmetric matrices, under similarity action of orthogonal group and the special orthogonal group respectively. We also study the Pfaffian case and the complex case. Inequalities are obtained.

Joint work with Mary Clair Thompson (Auburn University)

Revisiting a Permanent Conjecture on Positive Semidefinite Matrices

FUZHEN ZHANG, Nova Southeastern University, Fort Lauderdale, USA
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Thu 11:50, Auditorium

We will revisit the permanent conjecture $per(A \circ B) \leq per(A)per(B)$ with the maximizing matrix approach, where A and B are positive semidefinite matrices and $A \circ B$ is the Hadamard product of A and B.

The Equality Cases for the Inequalities of Oppenheim and Schur for Positive Semi-definite Matrices

XIAO-DONG ZHANG, Shanghai Jiao Tong University, P. R. China
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Thu 12:15, Auditorium

In matrix inequality theory, the inequalities of Oppenheim and Schur for positive semi-definite matrices are well known. In this talk, we investigate under which conditions the Hadamard product of two positive semi-definite matrices are singular. These results are used to give necessary and sufficient conditions for equality in the inequalities of Oppenheim and Schur for positive semi-definite matrices.

[1] R. B. Bapat and T. E. S. Ragharan, *Nonnegative Matrices and Applications,* Cambridge University Press, 1997.

[2] A. Oppenheim, Inequalities connected with definite Hermitian forms, *J. London Math. Soc.* 5 (1930), 114-119.

[3] X.-D. Zhang, and C.-X. Ding, The equality cases for the inequality of Oppenheim and Schur for positive semi-definite matrices, Czechoslovak Mathematical Journal, 59 (134) (2009), 197-206.

Joint work with Chang-Xing Ding (Shanghai Jiao Tong University)

Linear Algebra and Inverse Problems

Marco Donatelli, Università "Insubria", Como, Italy
James Nagy, Emory University, Atlanta, GA, USA

Inverse problems arise in many important applications, including medical imaging, microscopy, geophysics, and astrophysics. Because they often involve large scale, extremely ill-conditioned linear systems, linear algebra problems associated with inverse problems are extremely challenging to solve, both mathematically and computationally. Solution schemes require enforcing regularization, using for example prior information and/or by imposing constraints on the solution. In addition, matrix approximations and fast algorithms for structured matrices must be employed. The speakers in this minisymposium will report on recent research developments involving linear algebra aspects of inverse problems, including algorithms and other computational issues.

Nonsmooth/Smoothing Optimization Approaches to Structured Inverse Quadratic Eigenvalue Problems

Z.-J. Bai, Xiamen University, People's Republic of China
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Thu 16:45, Room Galilei

Structured inverse quadratic eigenvalue problems arise in the fields of structural dynamics, acoustics, electrical circuit simulation, fluid mechanics, etc. In this talk, we present some nonsmooth/smoothing optimization methods for solving structured inverse quadratic eigenvalue problems. The proposed algorithms are based on the recent developments in strong semismooth matrix-valued functions [1] and strong semismooth eigenvalues of symmetric matrices [2]. The global and locally fast convergence is established. Numerical experiments show the efficiency of the proposed methods.

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Bayesian Hypermodels for Inverse Problems

Johnathan M. Bardsley, University of Montana, USA
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Wed 12:15, Room Galilei

In this talk, I will discuss inverse problems in the context of Bayesian statistics, where the regularization function corresponds to the negative-log of the prior probability density. From the Bayesian perspective, the regularization parameter can be viewed as a hyper-parameter, i.e. as a random variable with some known distribution. Adding this element of uncertainty to the value of the regularization parameter is not only honest, it allows for increased flexibility. For example, one can sample from the posterior regularization parameter distribution, obtaining an empirical density (histogram) and hence confidence intervals for the regularization parameter. One can also allow for the regularization parameter to be spatially dependent (i.e. vector valued), which leads to adaptive methods and Bayesian learning. Numerical examples will be used to illustrate the various concepts.

A survey of scaled gradient projected methods for nonnegative image reconstruction

M. Bertero, University of Genova, Italy
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Wed 12:40, Room Galilei

In a regularization or Bayesian approach, the ill-posed problem of image reconstruction with nonnegativity constraint is reduced to the constrained minimization of a convex function. Implicit methods have a fast asymptotic convergence rate but require to solve a linear equation per iteration while explicit methods, with a slower convergence, require only matrix-vector multiplications. However, a recently proposed class of scaled gradient projection (SGP) method [1] can provide efficient algorithms, thus proposing these explicit methods as an interesting alternative to the implicit ones. Moreover, in the case of non-regularized minimization these methods still exhibit the *semi-convergence property.*

In this talk, after a general outline of the proposed SGP, we discuss a few applications. The first is to the problem of the nonnegative least-squares solution [3], the second to the denoising of images corrupted by Poisson noise [2] and the third to the non-regularized deblurring of Poisson data [1]. In such a case the SGP algorithm provides an acceleration of the standard EM method. Future applications to the regularized deblurring of Poisson data are also briefly discussed.

[1] S. Bonettini, R. Zanella and L. Zanni, A scaled gradient projection method for constrained image deblurring, Inverse Problems, 25, 015002 (23pp), 2009.
[2] R. Zanella, P. Boccacci, L. Zanni and M. Bertero, Efficient gradient projection methods for edge-preserving removal of Poisson noise, Inverse Problems, 25, 045010 (24pp), 2009.
[3] F. Benvenuto, R. Zanella, L. Zanni and M. Bertero, Nonnegative least-squares image deblurring: improved gradient projection approaches, Inverse Problems, 26, 025004 (18pp), 2010.

Joint work with S. Bonettini (University of Ferrara), R. Zanella and L. Zanni (University of Modena-Reggio Emilia), F. Benvenuto and P. Boccacci (University of Genova)

Designing Optimal Filters for Ill-Posed Inverse Problems

J. Chung, University of Maryland, College Park
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Thu 17:10, Room Galilei

Filtering methods are essential for computing reasonable solutions to ill-posed inverse problems. Without proper filtering, it is well known that small amounts of noise in the data may amplify, resulting in catastrophic errors in the solution. However, standard filtering methods such as Truncated-SVD and Tikhonov filtering may perform poorly for a given problem or application. In this paper, we are interested in designing optimal filters for a given operator of a given application. Utilizing techniques from stochastical and numerical optimization, we present a novel and efficient approach for constructing optimal filters based on minimizing the expected value of the mean square error estimates. Image deblurring is one application that relies heavily on robust filtering techniques, and numerical examples on testing data illustrate that our proposed filters perform consistently better than well established filtering methods.

Joint work with M. Chung (Emory University) D. P. O'Leary (University of Maryland, College Park)

Structured shift-variant imaging systems and invariant approximations via coordinate transformations

Claudio Estatico, Department of Mathematics and Computer Science, University of Cagliari, Italy
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Wed 11:50, Room Galilei

In the simplest Fredholm equations of the first kind arising in real applications, the integral kernel is shift-invariant, that is, the impulse response does not change as the object position is shifted. In image deblurring, this happens when exactly the same blur covers all the image domain. On the other hand, in the general case the shape of the impulse response might change as the object position is changed, that is, different regions of the image might be subjected to different blurs. These kinds of blurring models, termed as shift-variant, are much more involving since they require high numerical complexity in time and memory. However, many shift-variant integral kernels are intrinsically shift-invariant. We can call them as structured shift-variant. The well known main example is the rotational blur, which arises when the object rotates with respect to the imaging apparatus. Basically, although the blur changes with respect the object position (in particular, it is small close to and increases far from the center of the rotation), if the coordinate system is changed from Cartesian to Polar, then the integral kernel becomes explicitly shift-invariant. In this talk we analyze in a general and algebraic setting these kinds of structured shift-variant imaging systems. In this respect, we propose an algorithm for finding the coordinate transformation which allows a structured shift-variant PSF to become explicitly shift-invariant. The usage of the computed coordinate transformation will highly reduce the numerical complexity of the imaging system. Some numerical results related to a real application in External Vehicle Speed Control will end the talk.

Level set methods for the reconstruction of electrical conductivity by eddy current imaging

D. Fasino, University of Udine, Italy
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Thu 11:50, Room Galilei

We present a numerical method for solving an inverse problem for the 3D imaging of small, conductive inclusions in an insulating medium from exterior measurements. The method exploits a non-destructive technique based on eddy currents to analyze the response to the field of a probe coil placed at various positions and excited at different frequencies [1].

The computational problem consists of a large, distributed parameter estimation problem, having some peculiarities:

- The numerical evaluation of the forward map of the problem is a very expensive task;
- the sought solution is a piecewise constant function with a quite small support, and whose nonzero values are possibly known a priori;
- the continuous relaxation of the problem is a large, very under-specified, nonlinear problem.

Regularization is introduced as a sort of sparsity constraint on the (discretized) gradient of level set functions [2]. The talk illustrates various linear algebraic issues occurring in the numerical solution of this problem. The efficacy of the obtained method is substantiated by numerical simulations.

[1] A. Pirani, M. Ricci, R. Specogna, A. Tamburrino, F. Trevisan. Multi-frequency identification of defects in conducting media. *Inverse Problems* 24 (2008), 035011, 18 pp.
[2] A. DeCezaro, A. Leitão, X.-C. Tai. On multiple level-set regularization methods for inverse problems. *Inverse Problems* 25 (2009), 035004, 22 pp.

Joint work with R. Specogna (ruben.specogna@uniud.it, University of Udine, Italy) and F. Trevisan (trevisan@uniud.it, University of Udine, Italy)

Edge–Preserving Regularization in Color Image Reconstruction

I. Gerace, University of Perugia, Italy
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Wed 11:25, Room Galilei

Various color image inverse problems can be solved using a regularization technique. In this way the solution is defined as the argument of the minimum of an energy function, given by the sum of two terms. The first term is a consistency data term while the later one is related to the smoothness constraints. The design of the energy function is crucial for a proper image reconstruction. In this paper we focus on the construction of the smoothness term for an edge–preserving color image reconstruction.

Ideal images present intensity color discontinuities in correspondence of sharp color variations. By means of a duality theorem [1], we propose the use of a stabilizer that implicitly deals with line variables. These variable are related to the discontinuities in the intensity field. A correct estimation of the values of the line variables allows a more efficient image reconstruction. Many authors have noted as the high frequencies of the three RBG channels of a ideal image were very similar [2][3], so we propose to add to the smoothness term a new term related to the difference of the finite derivatives in different channels.

For the minimization of the energy function we propose a Graduated Non-Convex (GNC) technique and the experimental results confirm efficiency of the method.

[1] F. Martinelli, Regularization Techniques in Image and Signal Processing, PhD Thesis, University of Perugia, 2009.
[2] B.K. Gunturk, Y. Altunbasak, R.M. Mersereau, Color Plane Interpolation using Alternating Projections, IEEE Transactions on Image Processing, n. 9 vol. 11, pp. 997–1013, 2002.
[3] J. Mairal, M. Elad, G. Sapiro, Sparse Representation for Color Image Restoration, IEEE Transactions on Image Processing, n. 1 vol. 17, pp 53–69, 2008.

Sparse Approximate Inverse Preconditioning for Smoothing and Regularization

T. Huckle, Technical University Munich, Germany
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Thu 17:35, Room Galilei

We consider sparse approximate inverses for preconditioning iterative methods. Especially we are interested in applications where the iterative solver should reduce the error only in certain subspaces like in Multigrid or in ill-posed inverse problems. We derive two different methods to compute sparse approximate inverses with different behaviour on high frequency components and low frequency components. The new preconditioners lead to an improved smoothing property in Multigrid and to better reconstruction of the blurred data in ill-posed inverse problems.

Joint work with M. Sedlacek (Technical University Munich)

Edge Preserving Projection-based Regularization

Misha E. Kilmer, Tufts University, Medford, MA, USA

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Thu 11:25, Room Galilei

We present a projection-based regularization strategy and algorithm for retaining edges in a regularized solution. Our algorithm is suitable for large-scale discrete ill-posed problems arising from the discretization of Fredholm integral equations of the first kind; for example, image deblurring in two and three dimensions, the focus of our talk.

Our strategy avoids some of the pitfalls of many other well-known edge-preserving methods by making use of orthogonal decompositions/transforms in which components in the so-called noise and signal subspaces can be generated quickly. In determining the appropriate orthogonal transform, we exploit matrix structure as well as properties of the underlying continuous model. Numerical results show the promise of our approach.

Joint work with Per Christian Hansen (Technical University of Denmark), Donghui Chen (Tufts University)

Iterative methods for Tikhonov Regularization
Lothar Reichel, Kent State University, USA
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Thu 11:00, Room Galilei

The solution of linear discrete ill-posed problems is very sensitive to perturbations in the data. Tikhonov regularization is a popular approach to modifying these problems in order to make them less sensitive. We discuss iterative methods for the solution of large-scale Tikhonov-regularized problems with a general linear regularization operator.

Joint work with Hochstenbach, A Neuman, H. Sadok, F. Sgallari, and Q. Ye.

Multisplitting for Regularized Least Squares
Rosemary Renaut, Arizona State University
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Thu 12:15, Room Galilei

Least squares problems are one of the most often used numerical formulations in engineering. Many such problems lead to ill-posed systems of equations for which a solution may be found by introducing regularization. The use of multisplitting least squares, as originally introduced by Renaut for well-posed least squares problems, is extended to Tikhonov regularized large scale least squares problems. Regularization at both the global and subproblem level is considered, hence providing a means for multiple parameter regularization of large scale problems. Basic convergence results follow immediately from the original formulation. The iterative scheme to obtain the global solution uses repeated solves of local regularized systems each with a fixed system matrix but updated right hand side. Updates of the underlying Krylov subspace for the multiple right hand side system improve the efficiency of the local solver at each step. Numerical validation is presented for some simple one dimensional signal restoration simulations from the Regularization Toolbox of Hansen. The reconstruction of Shepp-Logan phantom data provides an example for a large scale problem. The use of local regularization parameters is also illustrated for a 1D restoration problem with variable noise in the signal. The implementation of the algorithm with GPUs for image restoration will also be discussed.

Joint work with Youzuo Lin (Arizona State University), Hongbin Guo (Arizona State University)

Image restoration by Tikhonov regularization based on generalized Krylov subspace methods
Fiorella Sgallari, Department of Mathematics-CIRAM, University of Bologna, Via Saragozza 8, 40123 Bologna, Italy.

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Wed 11:00, Room Galilei

We describe Tikhonov regularization of large linear discrete ill-posed problems with a regularization operator of general form and present an iterative scheme based on a generalized Krylov subspace method. This method simultaneously reduces both the matrix of the linear discrete ill-posed problem and the regularization operator. The reduced problem so obtained may be solved, e.g., with the aid of the singular value decomposition. Also, multiparameter Tikhonov regularization is discussed. Numerical results illustrate the promise of problem-oriented operator in image denoising and deblurring.

Joint work with L. Reichel (Kent State University) and Q. YE (University of Kentucky)

Max Algebras

Peter Butkovic, School of Mathematics The University of Birmingham, UK
Hans Schneider, Mathematics Department, University of Wisconsin, Madison, WI, USA

Max-algebra has existed as a form of linear algebra for almost half a century. We have seen a massive development in this area especially in the last 15 years. This is indicated by numerous papers published in leading journals, 5 books, and a good number of conferences or special sessions. This minisymposium provides state-of-the-art research presentations by established researchers in the field.

Representation of maxitive measures
M. Akian, INRIA Saclay–Île-de-France and CMAP, École Polytechnique, route de Saclay, 91128 Palaiseau Cedex, France
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Thu 12:15, Room Fermi

A maxitive measure is the analogue of a finitely additive measure or charge, in which the usual addition of reals is replaced by the supremum operation in a partially ordered set (poset). This notion was first introduced by Shilkret (1971), and reintroduced by many authors with different purposes such as capacity theory and large deviations, idempotent analysis and max-plus algebra, fuzzy set theory, optimisation, or fractal geometry.

A completely maxitive measure has a cardinal density, which means that there exists a map c such that the measure of any set is the supremum of c on that set. This property is related to the theory of residuation or Galois connections, or dualities.

We shall present and compare various representation results of this type. For instance, a countably maxitive measure on the topology of a separable metric space has a density (see [1] together with a max-plus linear form version shown by Kolokoltsov and Maslov (1988), see e.g. [2]). Barron, Cardaliaguet, and Jensen (2000) have shown a similar representation using essential suprema with respect to a usual positive measure, generalized in [3]. More generally, a maxitive measure can be decomposed as the supremum of a maxitive measure with density, and a residual maxitive measure that is null on compact sets [4].

[1] M. Akian. Densities of idempotent measures and large deviations. *Trans. Amer. Math. Soc.*, 351(11):4515–4543, 1999.

[2] V. Kolokoltsov and V. Maslov. *Idempotent analysis and applications*. Kluwer Acad. Publisher, 1997.

[3] P. Poncet. A note on two-valued possibility (σ-maxitive) measures and Mesiar's hypothesis. *Fuzzy Sets and Systems*, 158(16):1843–1845, 2007.

[4] P. Poncet. A decomposition theorem for maxitive measures, 2009. Accepted for publication in LAA, see also arXiv:0912.5178.

Joint work with P. Poncet

Tropical approximation of matrix eigenvalues

S. Gaubert, INRIA and CMAP, École Polytechnique, Palaiseau, France
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Mon 15:00, Auditorium

We establish several inequalities of log-majorization type, relating the moduli of the eigenvalues of a complex matrix with certain combinatorial objects, the tropical eigenvalues, which depend only on the moduli of the entries of the matrix. In nondegenerate cases, the orders of magnitude of the different eigenvalues of the complex matrix turn out to be given by the tropical eigenvalues. We use this information to perform a preprocessing (diagonal scaling) to improve the numerical accuracy of eigenvalue computations.

Joint work with M. Akian, M. Sharify

Characterization of non-strictly-monotone interval eigenvectors in max-min algebra

Martin Gavalec, University of Hradec Králové, Czech Republic
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Mon 16:45, Auditorium

The interval eigenproblem in max-min algebra for non-strictly-monotone eigenvectors is studied. For given real matrices $\underline{A}, \overline{A}$ of type (n, n), the matrix interval $\mathbf{A} = [\underline{A}, \overline{A}]$ is defined as the set of all matrices A, for which the inequalities $\underline{A} \leq A \leq \overline{A}$ hold true. For given real vectors $\underline{x}, \overline{x}$ of type $(n, 1)$ the vector interval $\mathbf{X} = [\underline{x}, \overline{x}]$ is the set of all vectors x with $\underline{x} \leq x \leq \overline{x}$. The interval eigenproblem $\mathbf{A} \otimes \mathbf{X} = \mathbf{X}$ is the problem of finding a solution to the equation $A \otimes x = x$ in max-min algebra, with additional conditions that the coefficient matrix A belongs to the given interval $\mathbf{A}$, and the eigenvector x belongs to $\mathbf{X}$. Six types of solvability of the interval eigenproblem are introduced, according to various combinations of quantifiers applied to $A \in \mathbf{A}$ and $x \in \mathbf{X}$. The characterization of all six types in the form of necessary and sufficient condition is given, with restriction to non-strictly-monotone eigenvectors. The conditions can be verified in polynomial time. All true implications between the solvability types are presented, and the false implications are illustrated by counter-examples.

Joint work with Ján Plavka (Technical University in Košice) and Hana Tomášková (University of Hradec Králové)

Tropical Rank and Beyond

Alexander Guterman, Moscow State University, Russia
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Mon 15:25, Auditorium

Rank functions over various classes of semirings are intensively investigated during the last decades. Among the other rank functions the following two are very important.

Let $(S, \oplus, \otimes)$ be a semiring, Σ_k be the permutation group on $\{1, \ldots, k\}$, $A_k \subset \Sigma_k$ be the subgroup of even permutations.

A matrix $A = [a_{ij}] \in M_k(S)$ is said to be *tropically singular* if there exists a subset $T \in \Sigma_k$ such that

$$\bigoplus_{\sigma \in T} a_{1\sigma(1)} \otimes \cdots \otimes a_{k\sigma(k)} = \bigoplus_{\sigma \in \Sigma_k \setminus T} a_{1\sigma(1)} \otimes \cdots \otimes a_{k\sigma(k)}.$$

Note that for tropical semirings this definition coincides with the classical one: the minimum in the permanent expression

$$\begin{aligned} \mathrm{per}(A) &:= \bigoplus_{\sigma \in S_k} a_{1\sigma(1)} \otimes \cdots \otimes a_{k\sigma(k)} \\ &= \min\{a_{1\sigma(1)} + \ldots + a_{k\sigma(k)} : \sigma \in S_k\} \end{aligned}$$

is attained at least twice.

Otherwise A is *tropically non-singular*.

Tropical rank of $M \in M_n(S)$ is the largest r such that M has a tropically non-singular $r \times r$ minor.

A matrix $A = [a_{ij}] \in M_k(R)$ is said to be *d-singular* if

$$\bigoplus_{\sigma \in A_k} a_{1\sigma(1)} \otimes \cdots \otimes a_{k\sigma(k)} = \bigoplus_{\sigma \in \Sigma_k \setminus A_k} a_{1\sigma(1)} \otimes \cdots \otimes a_{k\sigma(k)}.$$

Otherwise A is *d-non-singular*.

Determinantal rank of $M \in M_n(S)$ is the largest r such that M has a d-non-singular $r \times r$ minor.

This talk is devoted to our recent investigations of these two rank functions and their interrelations.

On the dual product and the dual residuation over idempotent semiring of intervals

L. Hardouin, University of Angers, France
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Mon 17:35, Auditorium

An idempotent semiring $\mathcal{S}$ can be endowed with a partial order relation defined as $a \preceq b \Leftrightarrow a \oplus b = b \Leftrightarrow a \wedge b = a$, in other words the sum operator $\oplus$ corresponds to the least upper bound of the set $\{a, b\}$. According to this order relation it is possible to obtain the greatest solution of equation $A \otimes X \preceq B$ where A, X and B are matrices of proper dimension and $(A \otimes X)_{ij} = \bigoplus_{k=1\ldots n} (a_{ik} \otimes x_{kj})$. The greatest solution is obtained by considering residuation theory and is practically given by $(X)_{kj} = \bigwedge_{i=1\ldots m} (a_{ik} \circ\!\!\!\backslash\, b_{ij})$, where $a_{ik} \circ\!\!\!\backslash\, b_{ij}$ is the greatest solution of the scalar equation $a_{ik} \otimes x_{kj} \preceq b_{ij}$. In this talk we will consider the dual matrix product $A \odot X$ defined as $(A \odot X)_{ij} = \bigwedge_{k=1\ldots n} (a_{ik} \otimes x_{kj})$, and the dual residuation to deal with computation of the smallest solution of inequality $A \odot X \succeq B$. Due to the lack of distributivity of operator $\otimes$ over the infimum operator $\wedge$, the existence of this unique solution is not always ensured. A sufficient condition is obtained when all the elements of the semiring admit an inverse, *i.e.* $\forall a \in \mathcal{S}$, $\exists b$ such that $a \otimes b = e$ where e is the identity element of the semiring. This condition is fulfilled

in (max-plus) algebra, and allows to deal with opposite semimodules in [1], but it is not the case in semirings of intervals such as introduced in [2]. Nevertheless a sufficient condition allowing to compute this smallest solution in this algebraic setting will be given.

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[2] L. Hardouin, B. Cottenceau, M. Lhommeau, and E. Le Corronc. Interval systems over idempotent semiring. Linear Algebra and its Applications, 431(5-7):855–862, August 2009.

Joint work with B. Cottenceau (University of Angers) and E. Le Corronc (University of Angers)

Nonlinear Markov games

V. Kolokoltsov, University of Warwick, UK
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Thu 11:00, Room Fermi

I will discuss a new class of stochastic games that I call nonlinear Markov games, as they arise as a (competitive) controlled version of nonlinear Markov processes (an emerging field of intensive research, see e.g. [1]-[3]. This class of games can model a variety of situation for economics and epidemics, statistical physics, and pursuit - evasion processes. Further discussion of this topic will be given in author's monograph [4].

Roughly speaking, a nonlinear Markov process is defined by the property that its future behavior depends on the past not only via its present position, but also its present distribution. A nonlinear Markov semigroup can be considered as a nonlinear deterministic dynamic system, though on a weird state space of measures (notwithstanding the fact that the specific structure of generators allows for a nontrivial stochastic interpretation of the evolution, which can be thought of as solving integral equation based on a path integral). Thus, as the stochastic control theory is a natural extension of the deterministic control, we are going to further extend it by turning back to deterministic control, but of measures. In particular, as introducing stochasticity in control destroys the max-plus linearity of the Bellman operator, the introduction of a nonlinear control can restore this linearity.

[1] V. Kolokoltsov. Nonlinear Markov Semigroups and Interacting Lévy Type Processes. Journ. Stat. Physics **126:3** (2007), 585-642.

[2] T.D. Frank. Nonlinear Markov processes. Phys. Lett. A **372:25** (2008), 4553-4555.

[3] M. Zak. Quantum Evolution as a Nonlinear Markov Process. Foundations of Physics Letters **15:3** (2002), 229-243.

[4] V. N. Kolokoltsov. Nonlinear Markov processes and kinetic equations. Monograph. To appear in Cambridge University Press 2010.

Supervisory control of a class of implicit systems

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Thu 11:25, Room Fermi

We aim at addressing the positive invariance of polytopic regions for implicit systems of the form

$$F\dot{w}(t) = \sum_{k=0}^{\nu} G_k x(t-\delta_k) \ . \tag{1}$$

Such a system arises from network models, in particular for a class of timed Petri nets, called linear nets [1]. A particularly important problem in this context is that of the supervisory control . For Petri nets [2], it consists of guarantying a bound on the variable $w(t)$, specified in terms of a polytopic region $\mathcal{P}(H,h) = \{w|Hw \leq h\}$. Thus, the supervisory control problem comes down to the positive invariance of the polytope $\mathcal{P}(H,h)$.

An important feature of system (1) is that in general, it is not regular, since the matrices F, and G_k, are rectangular. We aim at addressing the positive invariance of polytopic regions for such a system, trying to generalize the known results, that concern square regular systems [3].

[1] L. Libeaut, Sur l'utilisation des dioïdes pour la commande des systèmes à événements discrets, PhD thesis, Ecole Centrale de Nantes, France, 1996.

[2] M.V. Iordache and P.J. Antsaklis, A survey on the supervision of Petri nets, DES Workshop PN 2005, Miami, FL, June 21, 2005.

[3] S. Tarbouriech and E.B. Castelan, Positively invariant sets for singular discrete-time systems, Int. J. of Systems Science, vol.24, no.9, pp.1687-1705, 1993.

Joint work with J.J. Loiseau (IRCCyN, CNRS, Nantes, France)

An Idempotent Approach to Continuous-Time Stochastic Control Using Projection Operations

William M. McEneaney, University of California San Diego, USA
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Wed 11:00, Room Fermi

It is now well-known that many classes of deterministic control problems may be solved by max-plus or min-plus (more generally, idempotent) numerical methods. It has recently been discovered that idempotent methods are applicable to stochastic control and games. The methods are related to the curse-of-dimensionality-free idempotent methods for deterministic control. The first such methods for stochastic control were developed only for discrete-time problems. The key tools enabling their development were the idempotent distributive property and the fact that certain solution forms are retained through application of the dynamic programming semigroup operator. Using this technology, the value function can be propagated backwards with a representation as a pointwise minimum of quadratic or affine forms.

Here, we will remove the severe restriction to discrete-time problems. We obtain approximate solutions to the problems through repeated application of approximate backward dynamic programming operators. A generalization of the min-plus distributive property, applicable to continuum versions will be obtained. This will allow interchange of expectations over normal random variables with infimum operators. At each time-step, the solution will be represented as an infimum over a set of quadratic forms. Backward propagation is reduced to simple standard-sense linear algebraic operations for the coefficients in the representation. The difficulty with the approach is an extreme curse-of-complexity, wherein the number of terms in the min-plus expansion grows very rapidly as one propagates. The complexity growth will be attenuated via projection onto a lower dimensional min-plus subspace at each time step. At each step, one desires to project onto the optimal subspace relative to the solution approximation.

Joint work with Hidehiro Kaise (Nagoya University, Japan)

Max-plus linear systems
G. Merlet, Université de la Méditerranée, France
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Mon 15:50, Auditorium

Stochastic max-plus linear systems are defined as iterates of a random max-plus linear map. In this talk, we will present their long term behaviour and compare it to the one of positive linear systems. The first order results (law of large numbers [1,3]) are based on subadditivity and approximation by linear systems while the second order ones (Central limit theorems, [2]) rely on the geometric properties of action of the maps on the max-plus projective space and the approximation by sum of independant real variables.

[1] T. Bousch et Jean Mairesse, Finite-range topical functions and uniformly topical functions, Dynamical Systems 21, 1 (2006), pp. 73-114.
[2] G. Merlet, A central limit theorem for stochastic recursive sequences of topical operators, Ann. Appl. Probab. 17 (2007), no. 4, 1347-1361.
[3] G. Merlet, Cycle time of stochastic max-plus linear systems, Electronic Journal of Probability 13 (2008), Paper 12, 322-340.

Convex structures and separation in max-min (fuzzy) algebra
V. Nitica, West Chester University, USA
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Thu 11:50, Room Fermi

We present classification and separation results in max-min convexity. Separation by hyperplanes/halfspaces is a standard tool in convex geometry and its tropical (max-plus) analogue. Several separation results in max-min convex geometry are based on semispaces [1]. A counterexample to separation by hyperplanes in max-min convexity is shown in [2]. In the talk we answer the question which semispaces are hyperplanes and when it is possible to separate by hyperplanes in max-min convex geometry: a point can be separated from a convex set that does not contain it, if and only if the point belongs to the main diagonal. Further new separation results are presented, such as separation of a closed box from a max-min convex set by max-min semispaces. This can be regarded as an interval extension of the known separation results by semispaces [1]. We give a constructive proof of the separation in the case when the box satisfies a certain condition, and we show that the separation is never possible when the condition is not satisfied. These results hold in arbitrary finite dimension. We also study the separation of two max-min convex sets by a box and by a box and a semispace. These results hold only in the 2-dim case, and we provide counterexamples in the 3-dim case. The talk is based on [3] and [4].

[1] V. Nitica, The structure of max-min hyperplanes, Linear Algebra Appl., 432, pp. 402-429, 2010.
[2] V. Nitica and I. Singer, Contributions to max-min convex geometry. II. Semispaces and convex sets, Linear Algebra Appl., 428, pp. 2085–2115, 2008.
[3] V. Nitica and S. Sergeev, On hyperplanes and semispaces in max-min convex geometry, to appear in Kybernetika. arXiv:math/0910.0557
[4] V. Nitica and S. Sergeev, An interval version of separation by semispaces in max-min convexity, submitted to Linear Algebra Appl. arXiv:math/0910.0566

Joint work with S. Sergeev, University of Birmingham, UK

On the maximum cycle geometric mean
A. Peperko, University of Ljubljana, Slovenia
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Wed 12:15, Room Fermi

The maximum cycle geometric mean $\mu(A)$ of a $n \times n$ non-negative matrix A plays a role of the spectral radius in max algebra. We generalize the notion of the maximum circuit geometric mean to infinite non-negative matrices and provide several descriptions under certain conditions. In particular, we provide the max algebra description of $\mu(K)$, which provides connection to Bonsall's spectral radius and thus to max-eigenvalues.

If time allows, we will also consider some problems about submultiplicativity, subadditivity and the generalized spectral radius in max algebra.

Efficient algorithms for checking of the robustness and for computing the greatest eigenvector of a matrix in a fuzzy algebra
Ján Plavka, Technical University in Košice, Slovak Republic

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Mon 17:10, Auditorium

Let $(B, \leq)$ be a nonempty, bounded, linearly order set and $a \oplus b = \max(a, b)$, $a \otimes b = \min(a, b)$ for $a, b \in B$. A vector x is said to be a λ-eigenvector of a square matrix A if $A \otimes x = \lambda \otimes x$ for some $\lambda \in B$. We introduce some properties of the greatest λ-eigenvector of a given matrix A and in this context derive the $O(n^2 \log n)$ algorithm for computing the greatest λ-eigenvector [1]. A given matrix A is called (strongly) λ-robust if for every x the vector $A^k \otimes x$ is an (greatest) eigenvector of A for some natural number k. We present a characterization of λ-robust and strongly λ-robust matrices. As a consequence, an efficient algorithm for checking the λ-robustness and strong λ-robustness of a given matrix is introduced [2].

[1] M. Gavalec, J. Plavka, J. Polák: On the $O(n^2 \log n)$ algorithm for computing the greatest λ-eigenvector in fuzzy algebra (in preparation).

[2] J. Plavka: On the λ-robustness of matrices in a fuzzy algebra (submitted).

Joint work with Martin Gavalec (University of Hradec Králové, Czech Republic) and Ján Polák (Technical University in Košice, Slovak Republic)

Fundamental Traffic Diagrams : A Maxplus Point of View
J.-P. Quadrat, INRIA-Rocquencourt, France
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Wed 11:25, Room Fermi

Following Daganzo we discuss the variational formulation of the Lighthill-Witham-Richards equation describing the traffic on a road. First, we consider the case of a circular road with a very simple dynamics which is minplus linear. We extend it to the case of two roads with a junction with the right priority. The equation obtained is no more an Hamilton-Jacobi-Bellman equation. To study the eigenvalue problem extending the standard minplus one, we consider a space discretization of the equation. The discrete problem can be solved analytically, it gives the eigenvalue as function of the car density. The limit when the discretization step goes to zero gives a very simple formula. This eigenvalue gives a good approximation of what we call "the global fundamental traffic diagram" (the relation between the density and the average flow in the system). This global fundamental diagram must be distinguished from the standard fundamental diagram which is local, obtained empirically and, in the concave case, can be seen as an hamiltonian of a control problem.

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[2] J. Lighthill, J. B. Whitham: On kinetic waves: II A theory of traffic Flow on long crowded roads, Proc. Royal Society A229 p. 281-345, 1955.
[3] N. Farhi: Modélisation minplus et commande du trafic de villes régulière, thesis dissertation, University Paris 1 Panthéon - Sorbonne, 2008.

Joint work with N. Farhi (INRIA-Grenoble) & M. Goursat (INRIA-Rocquencourt)

The level set method for the two-sided eigenproblem in max-plus algebra

S. Sergeev, University of Birmingham, UK
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Wed 11:50, Room Fermi

We consider the max-plus analogue of the eigenproblem for matrix pencils, $Ax = \lambda Bx$. We show that the spectrum of (A, B) (i.e., the set of possible values of λ) is a finite union of intervals, which can be computed by a pseudo-polynomial number of calls to an oracle that computes the value of a mean payoff game. The proof relies on the introduction of a spectral function, which we interpret in terms of the least Chebyshev distance between Ax and λBx. The spectrum is obtained as the zero level set of this function.

Joint work with S. Gaubert (INRIA and École Polytechnique, France)

Optimization Problems under (max, min)-Linear Two-sided Equality Constraints

Karel Zimmermann, Charles University, Faculty of Mathematics and Physics
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Wed 12:40, Room Fermi

We consider the following optimization problem:

$$\text{minimize } f(x) \equiv \max_{j\in J} f_j(x_j)$$

subject to

$$\max_{j\in J}(a_{ij} \wedge x_j) = \max_{j\in J}(b_{ij} \wedge x_j) \ ,$$

$$\underline{x}_j \le x_j \le \overline{x}_j \, , \quad j \in J \ ,$$

where I, J are finite index sets, $f_j : R^1 \to R^1$ are continuous unimodal functions, $a_{ij}, b_{ij}, \underline{x}_j, \overline{x}_j$ are real numbers, and $\alpha \wedge \beta \equiv \min\{\alpha, \beta\}$ for any $\alpha, \beta \in R^1$.
An iteration method for solving the optimization problem is proposed. The method is based on a method for finding the maximum element of the set of feasible solutions of the given optimization problem combined with a bisection iterations. As a result an approximate solution of the given problem is obtained. Possibilities of applications of the considered class of problems are presented. The method is further used to approximate minimization of Lipschitzian objective functions under the given constraints. Extensions and generalizations of the presented results are briefly discussed.

Joint work with Martin Gavalec (University of Hradec Králové)

Generalized Inverses and Applications

Nieves Castro-Gonzalez, Universidad Politécnica de Madrid, Spain
Pedro Patricio, Departamento de Matematica, Universidade do Minho, Braga, Portugal

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Fredholm's method to solve a particular integral equation in 1903, was probably the first written work on generalized inverses. In 1906, Moore formulated the generalized inverse of a matrix in an algebraic setting, which was published in 1920, and in the thirties von Neumann used generalized inverses in his studies of continuous geometries and regular rings. Kaplansky and Penrose, in 1955, independently showed that the Moore "reciprocal inverse" could be represented by four equations, now known as Moore-Penrose equations. A big expansion of this area came in the fifties, when C.R. Rao and J. Chipman made use of the connection between generalized inverses, least squares and statistics. Generalized inverses, as we know them presently, cover a wide range of mathematical areas, such as matrix theory, operator theory, c*-algebras, semi-groups or rinRoom Fermigs. They appear in numerous applications that include areas such as linear estimation, differential and difference equations, Markov chains, graphics, cryptography, coding theory, incomplete data recovery and robotics. The aim of this mini-symposium, is to gather researchers involved in the study of generalized inverses and to encourage the exchange of ideas.

Analytic Perturbations of Generalized Inverses

Konstantin Avrachenkov, INRIA Sophia Antipolis, France
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Mon 12:15, Room Galilei

We investigate analytic perturbations of the reduced resolvent of a finite dimensional linear operator (also known as Drazin inverse in linear algebra literature). By analytic perturbations we mean the perturbed operator depends analytically on a perturbation parameter. Our approach is based on spectral theory of linear operators as well as on a new notion of group reduced resolvent. It allows to treat regular and singular perturbations in a unified framework. We produce an algorithm for computing the coefficients of the Laurent series of the perturbed reduced resolvent. In particular, the regular part coefficients can be calculated by simple recursive formulae. Finally, we apply these results to the perturbation analysis of Moore-Penrose generalized inverses.

Joint work with J. B. Lasserre (LAAS-CNRS)

On group invertibility and representations for the group inverse of partitioned matrices

N. Castro-González, Facultad de Informática, Universidad Politécnica de Madrid, 28660 Boadilla del Monte, Madrid,

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Mon 15:00, Room Galilei

In recent papers [1],[2], necessary and sufficient conditions were derived for a partitioned matrix to have several generalized inverses, including inner, reflexive and Moore-Penrose inverse, with Banachiewicz-Schur form. We recall that, if $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ and $A^=$ denotes a generalized inverse of A, then the Schur generalized complement of A in M is defined as $S = D - CA^=B$, and we say that the generalized inverse of M has the Banachiewicz-Schur form when it is expresible in the form

$$M^= = \begin{pmatrix} A^= + A^=BS^=CA^= & -A^=BS^= \\ -S^=CA^= & S^= \end{pmatrix}.$$

In this talk, firstly, we address the problem of developing conditions under which the Drazin inverse of a partitioned matrix can be obtained by a formula which involves the Banachiewicz-Schur form. Conditions for the existence of the group inverse of partitioned matrices satisfying the rank formula $\operatorname{rank}(M) = \operatorname{rank}(A^D) + \operatorname{rank}(S^D)$ are given. (Joint work with M.F. Martínez-Serrano).

Next, we study the group invertibility and give representations for the group inverse of a type of block matrices with applications in graph theory. (Joint work with J. Robles and J.Y. Vélez-Cerrada).

The research is partially supported by Project MTM2007-67232, "Ministerio de Educación y Ciencia" of Spain.

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Representations and additive properties of the Drazin inverse

D. Cvetković-Ilić, University of Niš, Serbia
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Tue 15:25, Room Galilei

The theory of Drazin inverses has seen a substantial growth over the past decades. Beside being of great theoretical interest it has found applications in many diverse areas, including statistics, numerical analysis, differential equations, Markov chains, population models, cryptography, control theory etc. One of the topics on the Drazin inverse that is of considerable interest concerns explicit representations for the Drazin inverse of a 2 × 2 block matrix and explicit representations for the Drazin inverse of the sum of two matrices. Until now, there has been no explicit formula for the Drazin inverse of $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ in terms of A^d and D^d with arbitrary A, B, C and D. In the recent years, the representation and characterization of Drazin inverses of matrices or operators on a Hilbert space have been considered by many authors.

Using an additive result for the Drazin inverse, we derive formulae for the Drazin inverse of a 2 × 2 block matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, under conditions weaker than those assumed in papers published before.

Also, we present some additive properties of the generalized Drazin inverse in a Banach algebra and find an explicit expression for the generalized Drazin inverse of the sum a + b in terms of a, a^d, b, b^d under various conditions.

A cancellation property of the Moore-Penrose inverse of triple products

Tobias Damm, Fachbereich Mathematik, TU Kaiserslautern, Kaiserslautern, Germany
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Mon 17:10, Room Galilei

We study the matrix equation

$$C(BXC)^\dagger B = X^\dagger \quad (*)$$

where $X^\dagger$ is the Moore-Penrose inverse, and we derive conditions for the consistency of (∗). Singular vectors of B and C are used to obtain all solutions . Applications to compliance matrices in molecular dynamics, to mixed reverse-order laws of generalized inverses and to weighted Moore-Penrose inverses are given.

Joint work with Harald Wimmer

New results concerning multiple reverse-order law

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Mon 17:35, Room Galilei

In this paper we present new results related to the mixed-type reverse order law for the Moore-Penrose inverse of the various products of multiple bounded Hilbert space operators. Some finite dimensional results are extended to infinite dimensional settings.

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Joint work with Dragan S. Djordjević (University of Niš)

On deriving the Drazin inverse of a modified matrix

E. Dopazo, Technical University of Madrid, Spain
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Mon 16:45, Room Galilei

Let A be an $n \times n$ complex matrix. The Drazin inverse of A is the unique matrix A^D satisfying the relations:

$$A^D A A^D = A^D, \quad A^D A = A^D A, \quad A^{k+1} A^D = A^k,$$

where k is the index of A. The concept of Drazin inverse plays an important role in various fields like Markov chains, singular differential and difference equations, iterative methods, etc.

A challenge in this area is to establish formulas for computing the Drazin inverse of a modified matrix in terms of the Drazin inverse of the original matrix. These formulas will be of great interest in various applications. They can be useful when the matrix can be expressed as the sum of a matrix with a convenient structure and an additive perturbation, in updating problems, etc.

This problem has been largely studied for invertible matrices. Starting from the well-known formula of Sherman-Morrison-Woodbury given for the regular case:

$$(A + UV^*)^{-1} = A^{-1} - A^{-1}U \left(I + V^* A^{-1} U\right)^{-1} V^* A^{-1},$$

where the matrix A and the Schur complement, $I + V^* A^{-1} U$, are invertible, an intensive research has been developed.

In the context of generalized inverses, some analogous formulas have been developed for the Moore-Penrose inverse and for the Drazin inverse under specific conditions. In this paper, we focus on deriving formulas for the Drazin inverse of a modified matrix in terms of the Drazin inverse of the original matrix and the generalized Schur complement, which extend results given in the literature.

This research has been partly supported by project MTM2007-67232, "Ministerio de Educación y Ciencia" of Spain.

Joint work with M.F. Martínez-Serrano (Technical University of Madrid)

Generalized inverses on the solution of the Toeplitz-pencil Conjecture

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Tue 15:00, Room Galilei

A 1981 conjecture by Bumby, Sontag, Sussmann, and Vasconcelos [1] says that the polynomial ring $\mathbb{C}[x]$ is a so called Feedback Cyclization (FC) ring. Two exceptional cases of that conjecture remained unsolved. In 2004 Schmale and Sharma [3] showed that one of these cases would follow from the truth of a simple looking conjecture they formulated for Toepliz matrices. In [2] the authors show that the Toeplitz pencil conjecture stated in [1] is equivalent to a conjecture for $n \times n$ Hankel pencils, and it is shown to be implied by another conjecture, which is called root conjecture, for matrices up to size 8×8. In this work we establish how the generalized inverse theory on matrices over rings can be applied to solve this problem.

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On the calculation of different type of generalized inverses for a rectangular matrix using the Kronecker canonical form

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Mon 15:25, Room Galilei

In several significant applications, in control and systems' modelling theory, the methodology of generalized inverses (for instance, the Drazin and the Moore-Penrose inverses) and the Matrix Pencil approach have been extensively used for the study of generalized (descriptor) linear systems with rectangular (or square) constant coefficients, see for instance [1-4]. In this new paper, we extend the recent results of [5]. Analytically, three main directions are discussed and presented:
(I) Using the complex Kronecker canonical form, we determine the $\{1, 2\}$-generalized inverse of a rectangular matrix.
(II) Under some interesting additional conditions, the Moore-Penrose inverse of a rectangular matrix is derived using also the matrix pencil approach.
(III) Finally, we prove - quite straightforwardly - that the is no connection between Drazin inverses and the Kronecker canonical form.

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Joint work with Athanasios A. Pantelous (Department of Mathematical Sciences, University of Liverpool) and Grigoris I. Kalogeropoulos (Department of Mathematics, University of Athens, Greece)

On Generalized Inverses and Green's Relations

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Tue 16:45, Room Galilei

We study generalized inverses on semigroups by means of Green's relations. We first define the notion of inverse along an element and study its properties. Then we show that the classical generalized inverses (group inverse, Drazin inverse and Moore-Penrose inverse) belong to this class. Finally, we prove continuity results for the inverse along an element, in topological rings and Banach algebras.

Recent results on generalized inverses

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Tue 17:10, Room Galilei

We present recent results on generalized inverse of elements in rings with involution. Particularly, the characterizations of partial isometries, EP and star-dagger elements in rings with involution are discussed. We also give several characterizations of Moore-Penrose-invertible normal and Hermitian elements in rings with involution and the proofs are based on ring theory only.

Joint work with D. S. Djordjević (University of Niš)

The generalized inverse of the rectangular Vandermonde matrix

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Mon 11:50, Room Galilei

A Vandermonde matrix is defined in terms of scalars

$\lambda_1, \lambda_2, ..., \lambda_m$ by

$$V_{nm} = V_n(\lambda_1, \lambda_2, ..., \lambda_m) = \begin{bmatrix} 1 & \lambda_1 & \cdots & \lambda_1^{n-1} \\ 1 & \lambda_2 & \cdots & \lambda_2^{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \lambda_m & \cdots & \lambda_m^{n-1} \end{bmatrix}.$$

This particular general family of matrices plays a significant role in different areas of mathematics and applied sciences, see [2], [4] etc. Following the existing literature, the most important applications of the Vandermonde matrix are appeared in approximation problems such as interpolation, least squares and moment problems.

Explicit formulas for solving Vandermonde systems and computing the inverse it are well known, see [1], [3-5] etc.

In this paper, we will discuss and present analytically the generalized inverse of the Rectangular Vandermonde matrix. This general class of Vandermonde matrix has been also appeared in control theory, see for instance [2] for more details.

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Joint work with Athanasios D. Karageorgos and Grigoris I. Kalogeropoulos (Department of Mathematics, University of Athens, Greece)

Additive Drazin inverses

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Mon 15:50, Room Galilei

We will address to the representation of the Drazin inverses, over a general (associative, with unity) ring, of the block matrix $M = \begin{bmatrix} a & c \\ b & 0 \end{bmatrix}$, in which the (2,2) block is zero. We aim for results in terms of "words" in the three blocks a, b and c, and their g-inverses, such as inner or Drazin inverses. The search for a formula for this Drazin inverse is closely related to the "additive problem" of finding the D-inverse of a sum $(a+b)^d$ in terms of words in a and b, and their g-inverses. As a special case, we shall examine the existence and representation of the group inverse of M.

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Joint work with R.E. Hartwig (North Carolina State University, USA)

Magic generalized inverses

George P. H. Styan, McGill University, Montréal (Québec), Canada
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Mon 11:00, Room Galilei

We consider singular fully-magic matrices in which the numbers in all the rows and columns and in the two main diagonals sum to the same number. Our interest focuses on such magic matrices for which the Moore–Penrose inverse and/or Drazin inverse may also be fully-magic, building on results in [1,2,3,4]. Examples include the matrices for some of the fully-magic squares considered by Heinrich Cornelius Agrippa von Nettesheim (1486–1535), Albrecht Dürer (1471–1528), and Bernard Frénicle de Bessy (c. 1605–1675).

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Joint work with Ka Lok Chu (Dawson College), S. W. Drury (McGill University) & Götz Trenkler (Universität Dortmund)

Nonnegative Drazin-projectors

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Tue 15:50, Room Galilei

In [1] a characterization of nonnegative matrices with nonnegative Drazin inverse was developed. Later, in [2] the authors gave a characterization of nonnegative matrices A such that $AA^{\#}$ is a nonnegative matrix, where $A^{\#}$ denotes the group inverse of the square matrix A. In the last paper only the case of matrices with index 1 was studied. The product $AA^{\#}$ will be called the group-projector of the matrix A.

In this work, firstly, a necessary and sufficient condition to obtain matrices A with nonnegative group projector is presented. The main contribution of this result is that the nonnegativity condition on the matrix A is removed. Next, the case of the matrix A with index greater than 1 is also analyzed. In this situation, an extended result for the nonnegativity of the Drazin-projector of A (that is, $AA^D \geq O$, where A^D represents the Drazin inverse of A) is obtained.

This paper has been partially supported by DGI grant MTM2007-64477 and by grant UPV number 2659.

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Joint work with Alicia Herrero (Instituto de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Spain) and Francisco J. Ramírez (Instituto Tecnológico de Santo Domingo, Dominican Republic)

Condition numbers for the LS and Tikhonov regularization of discrete ill-posed problems

Yimin Wei, Fudan University, China
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Mon 11:25, Room Galilei

One of the most successful methods for solving the linear least-squares (LS) problem $\min_x ||Ax - b||$ with a highly ill-conditioned or rank deficient coefficient matrix A is the method of Tikhonov regularization. In this talk, we derive the normwise, mixed and componentwise condition numbers and componentwise perturbation bounds for LS and the Tikhonov regularization. Our results are sharper than the known results. Some numerical examples are given to illustrate our results.

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Linear Algebra in Curves and Surfaces Modeling

Costanza Conti, Università di Firenze, Italy,
Carla Manni, Università di Roma Tor Vergata, Italy

Geometric modeling is the branch of applied mathematics devoted to methods and algorithms for mathematical description of shapes. Two-dimensional models are of crucial interest in design, technical drawing and computer typography, while three-dimensional models are central to computer-aided-geometric-design (CAGD) and computer-aided-manufacturing (CAM), and widely used in many applied technical fields such as civil and mechanical engineering, architecture, geology, medical image processing, scientific visualization, entertainment. Moreover, since CAGD methods are main ingredients in Isogeometric analysis – an emergent new paradigm for numerical treatment of PDEs which can be seen as a superset of FEMs – it turns out that geometric modeling acquires some relevance also in this area. The main goal of geometric modeling is to create and improve methods, and algorithms for curve and surface representations which is mainly achieved by means of suitable class of functions like splines, or refinable functions to which linear subdivision schemes are associated. For both, the manipulation and the analysis of such a class of functions, several tools of linear algebra play a crucial role like those suited for structured matrices, totally positive matrices, polynomial equations or computation of joint spectral radius. Therefore, aim of this mini-symposium is to gather scientists that, working on different aspects of curves ad surface modeling, face classical and new linear algebra problems and use linear algebra tools to move a step forward in their respective fields.

The 17–th Hilbert's problem and tight wavelet frames

Maria Charina, TU-Dortmund, Germany
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Tue 16:45, Room Fermi

The 17–th Hilbert's problem was solved in 1927 by Emil Artin. It says that each real, non–negative, multivariate polynomial can be written as a sum of squares of some rational functions. Hilbert in 1888 and Motzkin in 1960 showed that in general one cannot replace rational functions by polynomials in such polynomial representations. In the bivariate case, it is still an open question, the so–called sos problem, if any Laurent polynomial is a sum of squares of some other Laurent polynomials. In the dimention greater or equal to 3, this question has a negative answer as proved by Scheiderer in 1999. In this talk we show how to reduce the problem of constructing of tight wavelet frames, a certain redundant family of functions, to the pure algebraic sos problem. The optimization technique of the semi–definite programing allows us then to check the existence of the corresponding sos representations and to determine them, if they exist. Tight wavelet frames are of special interest as they play an important role in applications such as e.g. signal andimage processing.

Joint work with Joachim Stöckler (TU-Dortmund, Germany)

An algebraic approach to the construction of multichannel wavelet filters

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Tue 11:25, Room Fermi

In previous works [1,2], we proposed *full rank refinable functions* and *multichannel wavelets* as the proper wavelet tools for the analysis of functions which are *vector-valued.* In the orthogonal situation, the matrix filters associated to such functions have to satisfy the so-called *matrix quadrature mirror filter (QMF) equations*, which involve a large number of nonlinear conditions. In this talk we propose an efficient and constructive scheme for finding pairs of matrix solutions to QMF systems. The construction, which extends a procedure given in [3] to the full rank case, mainly makes use of *spectral factorization* techniques and of a matrix completion algorithm based on the resolution of generalized *Bezout identities.* Some examples illustrate the algorithm and the nature of the resulting matrix scaling functions/wavelets.

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Joint work with C. Conti (University of Firenze, Italy)

Approximate Implicitization and Approximate Null Spaces

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Tue 17:10, Room Fermi

The easy conversion of elementary curves and surfaces (lines, circles, ellipses, planes, spheres, cylinders, cones,...) to rational parametric and implicit representations is central in many algorithms used in CAD-systems. For rational Bézier and NURBS-surfaces no such easy conversion exists, a rational parametric surface of bi-degree (n_1, n_2) has in the general case an algebraic degree of $2n_1n_2$, giving the bi-cubic Beziér surface a degree 18 implicit representation. Essential to approximate implicitization is the combination of a rational parametric surface $\mathbf{p}(s,t)$, $(s,t) \in [0,1] \times [0,1]$, with the algebraic surface to be found $q(x,y,z,h) = 0$. The degree m of q, should satisfy $0 < m \leq 2n_1n_2$. The combination results in the following factorization, $q(\mathbf{p}(s,t)) = (\mathbf{Db})^T\mathbf{a}(s,t)$, where $\mathbf{b}$ contains the unknown coefficients of $\mathbf{q}$, and $\mathbf{a}(s,t)$ is an array that contains basis function represented in the tensor product Bernstein basis. Similar expressions exist for rational parametric curves and triangular Bézier surfaces. As the Bernstein basis is a partition of unity, and $(s,t) \in [0,1] \times [0,1]$ we have $|q(\mathbf{p}(s,t))| = \left\|(\mathbf{Db})^T\mathbf{a}(s,t)\right\|_2 \leq \left\|(\mathbf{Db})^T\right\|_2 \|\mathbf{a}(s,t)\|_2 \leq \left\|(\mathbf{Db})^T\right\|_2$. The smallest singular values and their respective coefficient vectors consequently represents alternative implicit approximations to $\mathbf{p}(s,t)$. If $m = 2n_1n_2$ we know that an exact solution exists and that the smallest singular value will be zero. The problem of finding an approximate algebraic representation of $\mathbf{p}(s,t)$ has been refomulated to a problem of finding an approximate null space of the matrix $\mathbf{D}$. One obvious choice is Singular Value Decomposition, however, alternative direct elimination methods also exist.

Joint work with Oliver Barrowclough and Jan B. Thomassen, SINTEF, Oslo, Norway

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Structured matrix methods for the construction of interpolatory subdivision masks

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Tue 11:50, Room Fermi

In this talk we discuss the general approach presented in [1] and [2] for the construction of interpolatory subdivision masks by relying upon polynomials and structured matrix computations.

[1]C. Conti, L. Gemignani, L. Romani, From symmetric subdivision masks of Hurwitz type to interpolatory subdivision masks, Linear Algebra Appl., 431, pp. 1971-1987, 2009.
[2] C. Conti, L. Gemignani, L. Romani, From approximating to interpolatory non stationary subdivision schemes with the same reproduction properties, Submitted.

Joint work with C. Conti (University of Florence), L. Romani (University of Milano-Bicocca)

Computing the joint spectral radius in some subdivison schemes.

N. Guglielmi, University of L'Aquila, Italy
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Mon 11:00, Room Fermi

In this talk I will consider the analysis of the joint spectral radius of infinite matrix sets arising in the convergence analysis of subdivision schemes [1]. This problem cannot be solved in the general case and presents serious difficulties also from the approximation perspective. Nevertheless it can be handled efficiently when the considered family depends linearly on its parameters. The main tool is the construction of a polyhedral invariant set for the family, which might be obtained in finite time under suitable assumptions. After recalling the main framework [2] and giving some theoretical results I will show some illustrative examples.

[1] N. Guglielmi, C. Manni and D. Vitale, On a class of C^2 Hermite interpolatory subdivision schemes, in preparation.
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Joint work with C. Manni and D. Vitale (University of Roma 2)

Nonnegative Subdivision Revisited

K. Jetter, Universität Hohenheim, Germany
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Tue 11:00, Room Fermi

Recent work by X. L. Zhou, see [3] and the references there, has settled a long-standing question of characterizing convergence of non-negative, univariate subdivision schemes. We relate some of these results to methods used in the analysis of non-homogeneous Markov processes. In particular, the convergence result in [1] (refering to even much older references) is a strong and so far less known basic theorem, from which convergence of nonnegative subdivision can be derived.

We will develope the main ideas and proofs following this approach through properties of stochastic matrices, and of products of families of such matrices. In particular, we will see that we can avoid the notion of the (in general uncomputable) joint spectral radius when dealing with nonnegative subdivision.

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Exact calculation of the JSR by depth first search on infinite trees

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Mon 11:50, Room Fermi

We report on our recent progress in computing precisely the joint spectral radius of two matrices. For many subdivision schemes that are relevant in praxis, we are able to specify the exact value of the associated joint spectral radius. Our method is based on a depth first search algorithm on an infinite binary tree whose knots in the k-th level are matrix products of length k. Using a colour coding, this infinite tree has a finite visualisation whose structure can be analysed.

[1] J.Hechler, B.Mößner, U.Reif, C^1-Continuity of the generalized four-point scheme, Linear Algebra and its Applications 430(2009) 3019-3029, Elsevier.

Joint work with Nicole Lehmann (Darmstadt University of Technology) and Ulrich Reif (Darmstadt University of Technology)

Parallel interactive shape modelling and deformation using subdivision surfaces

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Mon 12:15, Room Fermi

Subdivision surfaces provide a compact way to describe a smooth surface using a polygonal model. They are widely used in movie production, commercial modelers and game engines. In these contexts the goal is to enable real-time interactive editing, animation and rendering of smooth surface primitives. To achieve this goal we designed a parallel rendering pipeline which incorporates a special patch-based geometry shader for subdivision surface, integrated with a simple yet effective deformation framework for dynamic exact subdivision surfaces. The field of interactive shape deformation is a very challenging research field, since complex mathematical formulation have to be implemented in a sufficiently efficient and numerical robust manner to allow for interactive applications. Among the surface-based shape deformation techniques we discuss variational optimization and differential coordinates methods which modify differential surface properties instead of spatial coordinates. Linear deformation approaches present Inherent limitations which can be avoided by nonlinear techniques. However, a common drawback of such methods is that the computational effort and numerical robustness are strongly related to the complexity and quality of the surface tesselation.

Recent advances on the applications of totally nonnegative matrices to C.A.G.D.

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Tue 17:35, Room Fermi

It is well known that the bases whose collocation matrices are stochastic and totally nonnegative are the bases in C.A.G.D. with shape preserving properties. We present new applications of totally nonnegative matrices to C.A.G.D. We show that the progressive iteration property related to interpolatory curves has a close relationship with iterative methods applied to totally nonnegative matrices. We also comment some new optimal properties of bases, which are related with extremal properties of their corresponding collocation matrices.

Computing the joint spectral characteristics of large matrices.

V.Yu. Protasov, Moscow State University, Russia
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Mon 11:25, Room Fermi

The joint spectral characteristics of matrices such as the joint and lower spectral radii, the p-radius, the Lyapunov exponent, etc., have found many applications, in particular, in the study of refinement equations and subdivision schemes for curves and surfaces design. First we introduce a notion of a general self-similarity equations, whose special case is a refinement equation. Then we show that all joint spectral characteristics of matrices appear naturally as various regularity exponents of solutions of that equation. We consider several approaches for precise and approximate computation of that characteristics for large matrices (as they usually appear in the study of subdivision equations). The methods are based on the analysis of the corresponding extremal norms using tools of convex programming.

Algebraic conditions on non-stationary subdivision symbols for exponential reproduction

L. Romani, University of Milano-Bicocca, Italy
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Tue 12:15, Room Fermi

We present an accurate investigation of the algebraic conditions that the symbols of a convergent, binary, linear, non-stationary subdivision scheme should fulfill in order to reproduce spaces of exponential polynomials. A subdivision scheme is said to possess the property of reproducing exponential polynomials if, for any initial data uniformly sampled from some exponential polynomial function, the scheme yields the same function in the limit. The importance of this property is due to the fact that several functions obtained as combinations of exponential polynomials (such as conic sections, spirals or special trigonometric and hyperbolic functions) are of great interest in graphical and engineering applications. Since the space of exponential polynomials trivially includes standard polynomials, the results in this work extend the theory recently developed in [1] to the non-stationary context. As the symbol of the scheme changes from level to level and the parametrization plays a crucial role in this kind of study, the proofs of the non-stationary case are often significantly more difficult and intricate than in the stationary case, and much of the results previously obtained can not be straightforwardly generalized but require a complete reformulation. To illustrate the potentialities of these simple but very general algebraic conditions we will consider affine combinations of known subdivision symbols with the aim of creating new non-stationary subdivision schemes with enhanced reproduction properties.

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Joint work with C. Conti (University of Firenze)

Tensor Computations in Linear and Multilinear Algebra

Lek-Heng Lim, Berkeley, CA, USA
Eugene Tyrtyshnikov, RAS Moscow, Russia

Matrix computations with huge-size multilevel matrices, e.g. of order of 2 to power 100, are not easy to make feasible even with structure and supercomputers. However, the former seems much more essential for problems on that scale. Most important structure on that scale is related with separation of variables and eventually with tensors. Thus, successful matrix computations are becoming tensor computations. The purpose of this minisymposium is to present the state of the art in representation and approximation of tensors in higher dimensions. The accent is made on recent findings, in particular on the use of matrix methods for generalized unfolding matrices associated with tensors.

Approximation of High-Order Tensors by Partial Sampling: New Results and Algorithms

C. Caiafa, LABSP-Brain Science Institute, RIKEN, Japan
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Thu 11:00, Room B

Recently [1,2,3], a new formula was provided that allows one to reconstruct a *rank-*$(R_1, R_2, ..., R_N)$ *Tucker tensor* $\underline{\mathbf{Y}} \in \mathbb{R}^{I_1 \times I_2 \cdots \times I_N}$ from a subset of its entries which are determined by a selected subset of R_n indices in each mode $(n = 1, 2, ..., N)$. As a generalization of the column-row matrix decomposition (also known as **CUR** or "skeleton" decomposition), which approximates a matrix from a subset of its rows and columns, our result provides a new method for the approximation of a high dimensional ($N \geq 3$) tensor by using only the information contained in a subset of its *n-mode fibers* $(n = 1, 2, .., N)$. The proposed algorithm can be applied to the case of arbitrary number of dimensions ($N \geq 3$) and the indices are sequentially selected in an optimal way based on the previously selected ones. In this talk, we analyze and discuss the properties of this method in terms of the subspaces spanned by the *unfolding matrices* of the *subtensor* determined by the selected indices. We also discuss about its applications for signal processing where low dimensional signals are mapped to higher dimensional tensors and processed with tensor tools. Experimental results are shown to illustrate the properties and the potential of this method.

[1] C. Caiafa and A. Cichocki, Generalizing the Column-Row Matrix Decomposition to Multi-way Arrays, To appear.
[2] C. Caiafa and A. Cichocki, Reconstructing Matrices and Tensors from Few Vectors, Proc. NOLTA 2009, Oct. 18-21, 2009, Sapporo, Japan.
[3] C. Caiafa and A. Cichocki, Methods for Factorization and Approximation of Tensors by Partial Fiber Sampling, Cesar F. Caiafa and Andrzej Cichocki, Proc. CAMSAP 2009, Dec. 13-16, 2009, Aruba, Dutch Antilles.

Joint work with A. Cichocki (LABSP-Brain Science Institute, RIKEN)

Computing structured tensor decompositions in polynomial time

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Thu 11:25, Room B

Tensor decompositions permit to estimate in a deterministic way the parameters in a multi-linear model. Applications have been already pointed out in antenna array processing and digital communications [1], among others, and are extremely attractive provided some diversity at the receiver is available. In addition, they often involve structured factors. These deterministic techniques may be opposed to those based on cumulants, which require the decomposition of symmetric tensors [2]. More generally, the goal is to represent a function of three variables (or more) as a sum of functions whose variable separate.

As opposed to the widely used Alternating Least Squares algorithm, it is shown that non-iterative algorithms with polynomial complexity exist, when one or several factor matrices enjoy some structure, such as Toeplitz, Hankel, triangular, band, etc. Necessary conditions are first given, concerning dimensions, bandwidth, and rank [3]. Then sufficient conditions are provided, along with constructive algorithms, in the case of third order tensors. These algorithms require solving linear systems, and computing best rank-1 matrix approximations. Hence the overall complexity is polynomial if one admits that the latter rank-1 approximations also have a polynomial complexity.

[1] N. D. Sidiropoulos, G. B. Giannakis, and R. Bro, Blind Parafac receivers for DS-CDMA systems, IEEE Trans. on Sig. Proc., vol. 48, no. 3, pp. 810–823, Mar. 2000.
[2] P. Comon and G. Golub and L-H. Lim and B. Mourrain, Symmetric Tensors and Symmetric Tensor Rank, SIAM Journal on Matrix Analysis Appl., vol.30, no.3, Sept. 2008, pp.1254–1279.
[3] P. Comon and M. Sorensen and E. Tsigaridas, Decomposing tensors with structured matrix factors reduces to rank-1 approximations, Icassp, Dallas, March 14-19, 2010.

Joint work with M. Sorensen (I3S, University of Nice)

Optimization Problems in Contracted Tensor Networks

Mike Espig, Max-Planck-Institute for Mathematics in the Sciences, Germany
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Fri 16:45, Room B

In this talk we discuss a calculus of variations in arbitrary tensor representations with a special focus on contracted tensor networks and apply it to functionals of practical interest. The survey provides all necessary ingredients for applying minimization methods in a general setting. The important cases of target functionals which are linear and quadratic with respect to the tensor product are discussed, and combinations of these functionals are presented in detail. As an example, we consider the representation rank compression in tensor networks. For the numerical treatment, we introduce efficient methods. Furthermore, we demonstrate the rate of convergence in numerical tests.

Joint work with Wolfgang Hackbusch (Max-Planck-Institute for Mathematics in the Sciences) and Reinhold Schneider (Technical University Berlin)

Most Tensor Problems are NP Hard

Christopher Hillar, Mathematical Sciences Research Institute, Berkeley
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Thu 11:50 Room B

The idea that one might extend numerical linear algebra, the collection of matrix computational methods that form the workhorse of scientific and engineering computing, to numerical multilinear algebra, an analogous collection of tools involving hypermatrices/tensors, appears very promising and has attracted a lot of attention recently. We examine here the computational tractability of some core problems in numerical multilinear algebra. We show that tensor analogues of several standard problems that are readily computable in the matrix (i.e. 2-tensor) case are NP hard. Our list here includes: determining the feasibility of a system of bilinear equations, determining an eigenvalue, a singular value, or the spectral norm of a 3-tensor, determining a best rank-1 approximation to a 3-tensor, determining the rank of a 3-tensor over the real or complex numbers. Hence making tensor computations feasible is likely to be a challenge.

Joint work with Lek-Heng Lim (University of Berkeley)

Numerical solution of the Hartree-Fock equation in the multilevel tensor structured format

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Fri 17:10, Room B

We consider the numerical solution of the Hartree-Fock equation (nonlinear eigenvalue problem) by the novel tensor-structured methods based on tensor approximation of arising functions and operators represented on 3D $n \times n \times n$ Cartesian grid [1]. Tensor-structured techniques enable "agglomerated" computation of the three- and six- dimensional volume integrals [2], with complexity that scales linearly in the one-dimension grid size n. High accuracy is achieved due to the multigrid accelerated rank reduction algorithm for 3-rd order tensors which provides computation of the Hartree potential on large spacial grids, with $n \leq 10^4$, necessary to resolve multiple strong cusps in electron density [3]. The discrete nonlinear eigenvalue problem in 3D is solved iteratively by the multilevel tensor-truncated DIIS scheme on a sequence of refined grids with robust and fast convergence in a moderate number of iterations, uniformly in n, so that the overall computational cost also scales linearly in n. We present numerical illustrations for the all electron case of H_2O, and pseudopotential case of CH_4 and CH_3OH.

[1] B. N. Khoromskij, V. Khoromskaia, and H.-J. Flad. Numerical Solution of the Hartree-Fock Equation in the Multilevel Tensor-structured Format. Preprint MPI MiS 44/2009, Leipzig, July 2009, submitted.
[2] V. Khoromskaia. Computation of the Hartree-Fock Exchange in the Tensor-structured Format. Preprint MPI MiS 25/2009, Leipzig, June 2009.
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Joint work with H.-J.Flad and B. Khoromskij

Prospects of Quantics-TT Approximation in Scientific Computing

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Fri 11:00, Room B

We discuss the prospects of super-compressed tensor-structured quantics-TT data formats [1,3,5] in high dimensional numerical modeling. The respective multilinear algebra is based on the multi-folding or quantics representation of multidimensional data arrays [1,3]. Low rank tensor approximation via the TT-type dimension splitting scheme [2,4] leads to logarithmic complexity scaling in the volume size of a target N-d tensor. Numerical illustrations indicate that the quantics-TT tensor method has proved its value in application to various function related tensors arising in quantum chemistry and in the traditional FEM/BEM—the tool apparently works. In particular, this method can be applied in the framework of truncated iteration for solution the high dimensional elliptic/parabolic problems including stochastic PDEs.

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[2] I.V. Oseledets, and E.E. Tyrtyshnikov, *Breaking the Curse of Dimensionality, or How to Use SVD in Many Dimensions.* SIAM J. Sci. Comput., 31, 5(2009), 37-44-3759.
[3] I.V. Oseledets, *Tensors Inside of Matrices Give Logarithmic Complexity.* SIAM J. Matrix Anal., 2009, accepted.
[4] I.V. Oseledets, and E.E. Tyrtyshnikov, *TT-Cross Approximation for Multidimensional arrays.* Linear Algebra Appl., 432 (2010), 70-88.
[5] B.N. Khoromskij, I.V. Oseledets, *Quantic-TT approximation of elliptic solution operators in higher dimensions.* Preprint MPI MiS 79/2009, Leipzig 2009, submitted.

Tensor train and QTT decompositions for high-dimensional tensors

I. Oseledets, Institute of Numerical Mathematics, Russ. Acad. Sci.
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Fri 11:25, Room B

In this talk we develop the basic idea of tensor-train decomposition, which can be considered as natural extention of singular value decomposition to high dimensions. It does not suffer from the curse of dimensionality, and can be computed with the reliability and SVD. Basic subroutines are simple to implement and are available online. QTT decomposition opens a new application area for tensor decompositions – approximation of tensors of "physically small" dimension. It includes compact representation of functions on sufficiently fine tensor grids with 2^D points in each direction, leading to d log n complexity. When the tensor is in structured format, it is interesting to perform some operations with it. Some operations are very intuitive in the tensor-train format, however some are not. An important operation is finding maximal and minimal elements. An algorithm usin maximal-volume submatrices will be presented for finding maximal in modulus element in the TT format.

Joint work with E. E. Tyrtyshnikov (INM RAS), B. N. Khoromskij (MIS MPG)

Krylov subspace methods for tensor computations

Berkant Savas, The University of Texas at Austin
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Fri 11:50, Room B

In this talk we will present a few generalizations of matrix Krylov methods to tensors. The general objective is to obtain a rank-(p, q, r) approximation of a given $l \times m \times n$ tensor $\mathcal{A}$. The problem can be viewed as finding low dimensional signal subspaces associated to the different modes of $\mathcal{A}$. Krylov methods, similar to the matrix case, are particularly well suited for problems involving large and sparse tensors or for tensors that allow efficient multilinear tensor-times-vector multiplications. We will consider several different types of tensor in evaluating the proposed methods: (1) tensors with specified low ranks; (2) low rank tensors with added noise; and (3) large and sparse tensors. For a few special cases we will prove that our methods captures the true signal subspaces associated to the tensor within certain number of steps in the algorithm. For more general cases we propose an approach, based on the Krylov-Schur method for computing matrix eigenvalues, to improve the subspaces obtained from the tensor-Krylov procedures. Test results confirm the usefulness of the proposed methods for the given objective. The technical report [1] covers part of the topics discussed in this talk.

[1] B. Savas and L. Eldén, Krylov subspace methods for tensor computations, Technical Report LITH-MAT-R-2009-02-SE, 2009, Dept. of Math., Linköping University.

Joint work with Lars Eldén (Linköping University)

New algorithms for Tucker approximation with applications to multiplication of tensor-structured matrices and vectors

D. V. Savostyanov, Institute of Numerical Mathematics

RAS, Moscow
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Fri 12:15, Room B

New algorithms are proposed for Tucker approximation of tensors (multidimensional arrays) that are not given explicitly, but are defined by a tensor-by-vectors multiplication operation. As well as in matrix case, this framework applies to structured tensors, like sparse tensors, tensors with multilevel Toeplitz or Hankel structure and so on. We discuss the merits and drawbacks of minimal Krylov recursion [1] and suggest some possible optimisation for it. We also propose new approximation methods based on Wedderburn rank-reduction.

As an important application we consider approximate multiplication of d-dimensional matrices given as Tucker or canonical decomposition with the result being approximated in Tucker format with optimal values of ranks possible in the desired accuracy bound. Since mode sizes can be very large, the result should never appear as full array. Here we compare Krylov and Wedderburn approaches with previously studied independent factor filtering [3] and modified variable-rank Tucker-ALS procedure without a priori knowledge of ranks [2]. We also propose cheap initialization of Tucker-ALS using an intrinsic tensor structure of result. Numerical examples include structured evaluation of typical operators from Hartree-Fock/Kohn-Sham model, by means of Canonical-to-Tucker and Tucker-to-Tucker multiplication.

This work was supported by RFBR grants 08-01-00115, 09-01-12058, 10-01-00811 and RFBR/DFG grant 09-01-91332.

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Joint work with S. A. Goreinov, I. V. Oseledets (Institute of Numerical Mathematics RAS, Moscow)

Generalized Cross Approximation for 3d-tensors

Jan Schneider, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany
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Fri 17:35, Room B

In this talk we present a generalized version of the Cross Approximation for 3d-tensors. The given tensor $a \in \mathbb{R}^{n\times n\times n}$ is represented as a matrix of vectors and 2d adaptive Cross Approximation is applied in a nested way to get the tensor decomposition. The explicit formulas are derived for the vectors in the decomposition. The computational complexity of the proposed algorithm is shown to be linear in n.

Joint work with K. K. Naraparaju (MPI for Mathematics, Leipzig, Germany)

The future of tensor computations, or how to escape from the curse of dimensionality

E. Tyrtyshnikov, Insitute of Numerical Mathematics, Russ. Acad. Sci.
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Thu 12:15, Room B

Even "simple" cases in higher dimensions may require data elements as many as atoms in the universe. Structure in data in such cases is the key issue. However, existing tensor reprensentations of tensors (multilinear forms, multidimensional arrays) suffer from various drawbacks. We propose new tensor decompositions called TENSOR-TRAIN DECOMPOSITIONS and the corresponding numerical algorithms with then complexity linear in the number of axes. Applications include interpolation of multi-variate functions, computation of multi-dimensional integrals, solving PDEs, fast inversion of tensor structured matrices etc. The new algorithms appeared as recently as just in the beginning of 2009 and will certainly be leading to a new generation of numerical algorithms. For more details see http://pub.inm.ras.ru.

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[2] N.Zamarashkin, I.Oseledets, E.Tyrtyshnikov, The tensor structure of the inverse of a banded Toeplitz matrix, Doklady Mathematics, vol. 80, no. 2 (2009), pp. 669-670.
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[4] I. Oseledets, E. Tyrtyshnikov, TT-cross approximation for multidimensional arrays, Linear Algebra Appl., 432 (2010), pp. 70-88.

Joint work with I. Oseledets (INM RAS)

Linear Algebra in Quantum Information Theory

Vittorio Giovannetti,
Simone Severini,

The past two decades have witnessed a wide range of fundamental discoveries in quantum information science. These range from protocols revolutionizing public-key cryptography to novel algorithms and tools for communication, information processing, and simulation of physical systems. Even if the mathematical context of quantum information science is wide and multidisciplinary, linear algebra covers a major role, if not ubiquitous. In fact, by the standard formulation of quantum mechanics, physical states and their dynamics are both represented by matrices. The classification of quantum states, schemes for error-correcting codes, methods for allocating quantum resources, promising models of implementable computation, all need a vast number of linear algebraic notions and techniques. This minisymposium is intended as a workshop for strengthening communication between quantum information scientists and the linear algebra community. The minisymposium is a great occasion to present open problems and foster collaborations.

Characterization of circulant graphs having perfect state transfer

Milan Bašić, Faculty of Sciences and Mathematics, University of Niš, Serbia
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Fri 15:50, Auditorium

In this paper we answer the question of when circulant quantum spin networks with nearest-neighbor couplings can give perfect state transfer. The network is described by a circulant graph G, which is characterized by its circulant adjacency matrix A. Formally, we say that there exists a *perfect state transfer* (PST) between vertices $a, b \in V(G)$ if $|F(\tau)_{ab}| = 1$, for some positive real number τ, where $F(t) = \exp(\imath At)$. Saxena, Severini, Shparlinski in [3]proved that $|F(\tau)_{aa}| = 1$ for some $a \in V(G)$ and $\tau \in \mathbb{R}^+$ if and only if all eigenvalues of G are integer (that is, the graph is integral). The integral circulant graph $\mathrm{ICG}_n(D)$ has the vertex set $Z_n = \{0, 1, 2, \ldots, n-1\}$ and vertices a and b are adjacent if $\gcd(a - b, n) \in D$, where $D \subseteq \{d : d \mid n,\ 1 \le d < n\}$. These graphs are highly symmetric and have important applications in chemical graph theory. We show that $\mathrm{ICG}_n(D)$ has PST if and only if $n \in 4\mathbb{N}$ and $D = D_3 \cup D_2 \cup 2D_2 \cup 4D_2 \cup \{n/2^a\}$, where $D_3 \subseteq \{d : d \mid n,\ n/d \in 8\mathbb{N}\}$, $D_2 \subseteq \{d : d \mid n,\ n/d \in 8\mathbb{N} + 4\} \setminus \{n/4\}$ and $a \in \{1, 2\}$. We have thus answered the question of complete characterization of perfect state transfer in integral circulant graphs raised in [1]Furthermore, we also calculate perfect quantum communication distance (distance between vertices where PST occurs) and describe the spectra of integral circulant graphs having PST. For $n \in 4\mathbb{N}$ classes of $\mathrm{ICG}_n(D)$ such that PST exists between non-antipodal vertices are characterized. This answers a question posed by Godsil in [2]. We conclude by giving a closed form expression calculating the number of integral circulant graphs of a given order having PST.

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Indirect Hamiltonian Estimation

Daniel Burgarth, Imperial College London

Fri 15:00, Auditorium

It is well known that certain matrices with band structure are uniquely determined by their spectrum and only a few components of their eigenstates [1]. Recently, these methods have been applied to quantum spin models, demonstrating that Hamiltonian tomography can be performed indirectly [2-5]. From the perspective of quantum information this is useful because the standard process tomography is very inefficient. Further graph theoretical criteria were developed that tell us which components of the eigenstates need to be known in order to infer the full matrix (i.e., Hamiltonian) [4,6]. We review such methods and show how they can be generalized to arbitrary quadratic Hamiltonians of bosons or fermions [7].

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D. Driessche and H. V. D. Holst, to appear in Lin. Alg. App.
[7] D. Burgarth, K. Maruyama and F. Nori, in preparation.

Joint work with K. Maruyama and F. Nori (RIKEN, Japan)

A quantum algorithm for linear systems of equations

Aram Harrow, University of Bristol and Massachusetts Institute of Technology

Thu 16:45, Auditorium

Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector b, find a vector x such that $Ax = b$. We consider the case where one doesn't need to know the solution x itself, but rather an approximation of the expectation value of some operator associated with x, e.g., $x'Mx$ for some matrix M. In this case, when A is sparse, N by N and has condition number κ, classical algorithms can find x and estimate $x'Mx$ in $O(N\sqrt(\kappa))$ time. Here, we exhibit a quantum algorithm for this task that runs in $\mathrm{poly}(\log N, \kappa)$ time, an exponential improvement over the best classical algorithm. This talk is based on arXiv:0811.3171

Joint work with Avinatan Hassidim and Seth Lloyd

Higher-order functions in Quantum Theory

Paolo Perinotti, University of Pavia

Thu 17:10, Auditorium

I will introduce the theory of higher-order functions in Quantum Theory. The main theorems and their application to circuit optimisation problems will be reviewed. I will show how the theory of Quantum Combs provides a proper framework to describe and optimise all quantum algorithms explored so far, but does not exhaust the theory of Quantum Computation. I will exhibit the primitive of quantum switch, and show the problems that such a simple task poses to the characterisation of the full hierarchy of higher order maps.

Joint work with G. Chiribella and G. M. D'Ariano

Perfect state transfer in integral circulant graphs

Marko Petkovic, University of Nis

Fri 15:25, Auditorium

The existence of perfect state transfer (PST) in quantum spin networks has been proposed by Christandl et. al. (2004) where they considered simple paths as a potential candidates for the network topology. Furthermore, Saxena, Severini and Shparlinski (2007) considered the networks based on circulant graphs. We extend the result of Saxena, Severini and Shparlinski (2007) and give the simple condition for characterizing all integral circulant graphs (ICGs) having the PST in terms of its eigenvalues. In this paper, it is proven that there exist integral circulant graph with n vertices having perfect state transfer if and only if $4 \mid n$. There are found several classes of integral circulant graphs having perfect state transfer for values of n divisible by 4. Moreover we proved the non-existence of PST for several other classes of integral circulant graphs whose order is divisible by 4. These classes cover the class of graphs where divisor set contains exactly two elements. Obtained results provides the first of two steps in solving the general problem: Which integral circulant graphs have PST?

Complex Hadamard matrices and combinatorial designs

Ferenc Szollosi, Central European University, Budapest

Thu 15:50, Auditorium

In the first part of the talk we present a design theoretical approach to construct new, previously unknown complex Hadamard matrices of prime orders. Our methods generalize and extend the earlier results of de la Harpe–Jones [1] and Munemasa–Watatani [2] and offer a theoretical explanation for the existence of some sporadic examples of complex Hadamard matrices in the existing literature. In the second part we obtain equiangular tight frames of square orders from complex Hadamard matrices settling a recent question of Bodmann et al [3].

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Continuous families of complex (generalized) Hadamard matrices

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Thu 15:25, Auditorium

An $N \times N$ complex Hadamard matrix is a matrix with orthogonal rows and columns (after rescaling a unitary) with all entries of modulus equal to one. The search for these is a special case of the search for unitary preimages of doubly stochastic matrices, which is of importance in particle physics. Complex Hadamard matrices have additional applications of their own, in particular in quantum information theory.

In this talk we present the currently known classification of complex Hadamard matrices of small size, which includes various continuous families. Also, results and hypotheses concerning construction of so called affine Hadamard families will be presented. By affine we mean a family where matrices of parametrizing phases form a linear subspace of the space of all real $N \times N$ matrices.

Zero-error communication via quantum channels, non-commutative graphs and a quantum Lovasz theta function

Andreas Winter, University of Bristol and National University of Singapore

Thu 17:35, Auditorium

We present the quantum channel version of Shannon's zero-error capacity problem. Motivated by recent progress on this question, we propose to consider a certain operator space as the quantum generalisation of the adjacency matrix, in terms of which the plain, quantum and entanglement-assisted capacity can be formulated, and for which we show some new basic properties. Most importantly, we define a quantum version of Lovasz' famous theta function, as the norm-completion (or stabilisation) of a "naive" generalisation of theta. We go on to show that this function upper bounds the number of entanglement-assisted zero-error messages, that it is given by a semidefinite programme, whose dual we write down explicitly, and that it is multiplicative with respect to the natural (strong) graph product. We explore various other properties of the new quantity, which reduces to Lovasz' original theta in the classical case, give several applications, and propose to study the operator spaces associated to channels as "non-commutative graphs", using the language of Hilbert modules. The talk is based on arXiv:1002.2514v2 [quant-ph]

Joint work with Runyao Duan, Simone Severini

Generalized numerical range as a versatile tool in the theory of quantum information

Karol Zyczkowski, Jagiellonian University, and Center for Theoretical Physics, Warsaw

Thu 15:00, Auditorium

We study operators acting on a composite Hilbert space and investigate their product numerical range, product spectral radius and product C–spectral radius. For any Hermitian operator X acting on a bi-partite Hilbert space its product numerical range is formed by the set of all possible expectation values of X among pure product states, $\langle\phi| \otimes \langle\psi|X|\psi\rangle \otimes |\phi\rangle$. Concrete bounds for the product numerical range for Hermitian operators are derived. Product numerical range of a non-Hermitian operator forms a subset of the standard numerical range. While the latter set is convex, the product range needs not to be convex nor simply connected. Product numerical range of a tensor product is equal to the Minkowski product of numerical ranges of individual factors. As an exemplary application of these algebraic tools in the theory of quantum information we study block positive matrices and entanglement witnesses. Furthermore, we apply product numerical range to solve the problem of local distinguishability of a family of two unitary gates. Product C–spectral radius is useful for finding local fidelity between two states of a composite system, while higher order product numerical range can be used to design local quantum dark spaces and local error correction codes.

Matrix Means

Jimmie Lawson, Louisiana State University
Yongdo Lim, Kyungpook National University, Taegu, Korea

The theory of matrix and operator means is currently an active area of research. Investigations include the theoretical study of such means, various axiomatic and variational descriptions and characterizations, computational algorithms for their approximation, geometric interpretations and connections, and applications in a variety of settings. Recent advances include various approaches to define, study, and compute a variety of multivariable means. Applications include derivations of matrix and operator inequalities, finding closed formulas and approximating algorithms for the solution of symmetric and other matrix equations. Another active direction of research is the employing of means for the purpose of averaging, with applications including the averaging of data given in matrix form.

Higher order geometric mean equations based on monotone and jointly homogeneous maps

EUNKYUNG AHN, Kyungpook National University, Korea
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Fri 12:15, Auditorium

We consider the nonlinear equations based on monotone and jointly homogeneous maps on the convex cone of positive definite matrices. We'll derive the uniqueness and existence of positive definite solution by using Thompson's part metric and that the corresponding solution map is again monotone and jointly homogeneous. Let $\Omega = \Omega(k)$ be the convex cone of $k \times k$ positive definite matrices. We first show that for monotone and jointly homogeneous mappings $g : \Omega^n \to \Omega$ and $h_i : \Omega^2 \to \Omega$, the equation

$$x = g(h_1(a_1, x), h_2(a_2, x), \ldots, h_n(a_n, x))$$

has a unique solution in Ω if $\sum_{i=1}^n w_i\alpha_i \in [0, 1)$. Here, a map $g : \Omega^n \to \Omega$ is $\mathbf{w} = (w_1, w_2, \ldots, w_n)$-jointly homogeneous if $g(t_1a_1, t_2a_2, \ldots, t_na_n) = t_1^{w_1}t_2^{w_2}\cdots t_n^{w_n}g(a_1, a_2, \ldots, a_n)$ for all $t_i > 0$ and $a_i \in \Omega$. Also $h : \Omega^2 \to \Omega$ is α-homogeneous if it is $(1 - \alpha, \alpha)$-jointly homogeneous. We further show that if $\sum_{i=1}^n w_i = 1$ and $\alpha_i = \alpha$ for all i, then the solution map varying over $(a_1, a_2, \ldots, a_n) \in \Omega^n$ is again order preserving and $\mathbf{w}$-jointly homogeneous. We apply our results to high order geometric mean equations of positive definite matrices.

Matrix Means in a Euclidean setting

KOENRAAD M.R. AUDENAERT, Royal Holloway, University of London, UK
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Wed 12:15, Auditorium

Matrix means are defined for positive semidefinite matrices, and as such are usually studied from the viewpoint of Riemannian geometry, with the set of positive definite matrices being a differentiable Riemannian manifold. In this paper, a completely different approach is taken, inspired by certain practical problems in quantum state reconstruction. To wit, we regard the set of positive definite matrices as a subset of the set of Hermitian matrices, equipped with the Hilbert-Schmidt (HS) inner product, i.e. as a real Euclidean space.

We investigate which matrix norms obey the requirement that 'their value should lie inbetween the values of their arguments'. To make sense of the term 'inbetween', we consider a) the HS distance, and b) the angle between matrices $\cos\theta(A, B) = Tr(A^*B)/\sqrt{Tr(A^*A)Tr(B^*B)}$. We define a matrix mean $C = \mu(A, B)$ to lie within A and B w.r.t. HS distance if and only if neither the distance between A and C, nor the distance between B and C exceed the distance between A and B. Similarly, we define a matrix mean to lie within A and B w.r.t. angles if and only if neither the angle between A and C, nor the angle between B and C exceed the angle between A and B.

It turns out that many matrix means do not satisfy 'inbetweenness' in neither sense. Here we show that the inbetweenness condition is satisfied by the power means and the Heinz means, for distances as well as for angles.

Interpolation, geometric mean and matrix Chebyshev inequalities

JEAN-CHRISTOPHE BOURIN, Université de Franche-Comté, France
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Wed 11:25, Auditorium

The geometric mean of positive definite matrices may be defined via complex interpolation. This approach leads to to simple proofs of Ando-Hiai and Furuta inequalities. We then show how these inequalities are used to obtain new inequalities for positive linear maps, regarded as asymmetric versions of Kadison and Choi inequalities. This talk is based on a joint paper with Éric Ricard.

Operator inequalities related to weighted geometric means

MASATOSHI FUJII, Osaka Kyoiku University, Japan
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Tue 16:45, Auditorium

The geometric mean $A \sharp B$ for positive operators A and B is given by the unique positive solution of the operator equation $XA^{-1}X = B$. That is,

$$A \sharp B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}}.$$

By virtue of the Kubo-Ando theory, it is generalized to weighted geometric means as follows: For $\alpha \in [0, 1]$

$$A \sharp_\alpha B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\alpha}A^{\frac{1}{2}}.$$

It corresponds to the Löwner-Heinz inequality:

$$A \geq B \geq 0 \implies A^\alpha \geq B^\alpha.$$

There are many useful operator inequalities related to this. A typical example is the Ando-Hiai inequality (AH):

$$A \sharp_\alpha B \leq 1 \implies A^r \sharp_\alpha B^r \leq 1 \quad \text{for} \quad r \geq 1.$$

In this talk, we discuss generalizations of (AH) and relations among obtained inequalities. Our basic inequality is as follows:

If $\log A \geq \log B$ for $A, B > 0$, then

$$A^{-r} \sharp_{\frac{r}{p+r}} B^p \leq 1$$

holds for $p, r \geq 0$.

Operator equations via an order preserving operator inequality

TAKAYUKI FURUTA, Tokyo University of Science, Japan
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Tue 15:50, Auditorium

A capital letter means a bounded linear operator on a Hilbert space. We obtained the following order preserving operator inequality closely associated with matrix means:

Theorem A. If $A \geq B \geq 0$, then the following (i) and (ii) hold for $p \geq 1$ and $r \geq 0$;

(i) $(B^{\frac{r}{2}}A^pB^{\frac{r}{2}})^{\frac{1+r}{p+r}} \geq B^{1+r}$ and (ii) $A^{1+r} \geq (A^{\frac{r}{2}}B^pA^{\frac{r}{2}})^{\frac{1+r}{p+r}}$.

Let A be a positive definite operator and B be a self-adjoint operator. We discuss the existence of positive semidefinite solutions of the Lyapunov type operator equation

$$\sum_{j=1}^{n} A^{n-j}XA^{j-1} = B$$

via Theorem A and by using the solutions we give concrete and recordable examples of positive semidefinite matrices as positive semidefinite solutions of some matrix equations.

The tracial geometric mean in several variables and related trace inequalities

F. Hansen, University of Copenhagen, Denmark
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Tue 15:00, Auditorium

We introduce the tracial geometric mean of several operator variables as a generalization of the geometric mean for tuples of positive numbers. It possesses a number of attractive properties, including monotonicity and concavity in the operator variables. The non-commutative Hardy inequality is used to obtain a generalization of Carleman's inequality. Other related trace inequalities are given.

Operator log-convex functions and operator means

F. Hiai, Tohoku University, Japan
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Tue 15:25, Auditorium

We were motivated by the question to determine $\alpha \in \mathbb{R}$ for which the functional $\log \omega(A^\alpha)$ is convex in positive operators A for any positive linear functional ω. In the course of settling the question, we arrived at the idea to characterize continuous nonnegative functions f on $(0, \infty)$ for which the operator inequality $f\big(\frac{A+B}{2}\big) \le f(A) \# f(B)$ holds for positive operators A and B, where $A \# B$ is the geometric mean. This inequality was formerly considered by Aujla, Rawla and Vasudeva as a matrix/operator version of log-convex functions. In fact, it is natural to say that a function f satisfying the above inequality is operator log-convex, since the numerical inequality $f\big(\frac{a+b}{2}\big) \le \sqrt{f(a)f(b)}$ for $a, b > 0$ means the convexity of $\log f$ and the geometric mean $\#$ is the most standard operator version of geometric mean. We show that a continuous nonnegative function f on $(0, \infty)$ is operator log-convex if and only if it is operator monotone decreasing, and furthermore present several equivalent conditions related to operator means for the operator log-convexity. The operator log-concavity counterpart is also considered.

Joint work with T. Ando (Hokkaido University)

Recent researches on generalized Furuta-type operator functions

M. Ito, Maebashi Institute of Technology, Japan
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Wed 11:50, Auditorium

In what follows, A and B are positive (semidefinite) operators on a Hilbert space, and $A \ge 0$ (resp. $A > 0$) denotes that A is a positive (resp. strictly positive) operator.

Furuta inequality *"$A \ge B \ge 0$ ensures $A^{1+r} \ge (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{1+r}{p+r}}$ for $p \ge 1$ and $r \ge 0$"* is established in 1987, and also Furuta showed its generalization (called grand Furuta inequality) in 1995 as follows: *If $A \ge B \ge 0$ with $A > 0$, then for each $t \in [0,1]$ and $p \ge 1$,*

$$F(r,s) = A^{\frac{-r}{2}} \{A^{\frac{r}{2}} (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^s A^{\frac{r}{2}}\}^{\frac{1-t+r}{(p-t)s+r}} A^{\frac{-r}{2}} \tag{1}$$

is decreasing for $r \ge t$ and $s \ge 1$, and also for each $t \in [0,1]$ and $p \ge 1$,

$$A^{1-t+r} \ge \{A^{\frac{r}{2}} (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^s A^{\frac{r}{2}}\}^{\frac{1-t+r}{(p-t)s+r}}$$

holds for $r \ge t$ and $s \ge 1$. We remark that grand Furuta inequality is interpolating Furuta inequality and Ando-Hiai inequality which is equivalent to the main result of log majorization. Very recently, Furuta obtained a further extension of grand Furuta inequality (we call this FGF inequality here).

α-Power mean $\sharp_\alpha$ for $\alpha \in [0,1]$ is defined by $A \sharp_\alpha B = A^{\frac{1}{2}} (A^{\frac{-1}{2}} B A^{\frac{-1}{2}})^\alpha A^{\frac{1}{2}}$ for $A > 0$ and $B \ge 0$. It is known that α-power mean is very usful for investigating Furuta inequality and its generalizations. We can express (1) by (1') with α-power mean as follows:

$$\begin{aligned} F(r,s) &= A^{-r} \sharp_{\frac{1-t+r}{(p-t)s+r}} (A^{\frac{-t}{2}} B^p A^{\frac{-t}{2}})^s \\ &= A^{\frac{-t}{2}} \{A^{-\gamma} \sharp_{\frac{1+\gamma}{\beta+\gamma}} (A^t \natural_{\frac{\beta-t}{p-t}} B^p)\} A^{\frac{-t}{2}} \qquad (1') \\ &= A^{\frac{-t}{2}} \hat{F}(\beta,\gamma) A^{\frac{-t}{2}}, \end{aligned}$$

where $\beta = (p-t)s + t$, $\gamma = r - t$ and $A \natural_s B = A^{\frac{1}{2}} (A^{\frac{-1}{2}} B A^{\frac{-1}{2}})^s A^{\frac{1}{2}}$ for a real number s. (If $s \in [0,1]$, then $\natural_s = \sharp_s$.)

In this talk, firstly we shall discuss complementary inequalities and related results to generalized Ando-Hiai inequality and a generalized Furuta-type operator function. Secondly we shall obtain a more precise and clear expression of FGF inequality by considering a mean theoretic proof of grand Furuta inequality.

Joint work with E. Kamei (Maebashi Institute of Technology)

The Weighted Multivariable AGH-Mean

Se-Jong Kim, Louisiana State University, Baton Rouge, USA

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Fri 11:00, Auditorium

In this presentation we consider a weighted mean arising as the geometric mean of the weighted arithmetic and harmonic n-means of positive definite matrices, what we call the AGH-mean. This mean is readily computable and exhibits a variety of other desirable properties, which we describe. We show that it also has nice variational characterizations. We also show that it generalizes in a straightforward fashion to a one-parameter family of weighted means and that many of its properties carry over to this generalization.

Joint work with J. Lawson (Louisiana State U.), Y. Lim (Kyungpook National U.)

Weighted Ando-Li-Mathias Geometric Means

Hosoo Lee, Kyungpook National University, Korea
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Fri 11:50, Auditorium

In [1], Ando-Li-Mathias proposed a successful definition for geometric means of several positive definite matrices. We propose a higher order weighted geometric mean based on the Ando-Li-Mathias symmetrization procedure.

For positive real numbers s and t, $G(s,t;A,B)$ is defined by $G(s,t;A,B) = A \#_{\frac{t}{s+t}} B$. A weighted geometric mean $G(t_1, t_2, \ldots, t_n; A_1, A_2, \ldots, A_n)$ of positive definite matrices $A_1, A_2, \ldots, A_n$ and positive real numbers $t_1, t_2, \ldots, t_n$ is defined by induction as follows: Assume that the weighted geometric mean of any $(n-1)-$ tuple of matrices is defined. Let

$$G((t_j)_{j\neq i};(A_j)_{j\neq i}) = G(t_1,\ldots,t_{i-1},t_{i+1},\ldots,t_n; A_1,\ldots,A_{i-1},A_{i+1},\ldots,A_n)$$

and let $A_i^{(1)} = A_i$ and $A_i^{(r+1)} = G((t_j)_{j\neq i};(A_j^{(r)})_{j\neq i})$. Then the sequences $A_i^{(r)}$ converge to a common limit, denoted by $G(t_1,\ldots,t_n;A_1,\ldots,A_n) = \lim_{r\to\infty} A_i^{(r)}$.

We show that the weighted mean satisfies the properties given by Ando-Li-Mathias in a weighted version: consistency with scalars, joint homogeneity, permutation invariance, monotonicity, continuity, invariance under the congruence and inversion, joint concavity, self-duality, determinant identity and arithmetic-geometric-harmonic means inequality.

[1] T. Ando, C.K. Li and R. Mathias, Geometric means, Linear Algebra Appl., 385 (2004), 305-334.

Joint work with Yongdo Lim (Kyungpook National University) and T. Yamazaki (Kanagawa University)

Weighted Bini-Meini-Poloni Geometric Means

Yongdo Lim, Kyungpook National University, Korea
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Tue 17:35, Auditorium

Taking a weighted version of Bini-Meini-Poloni symmetrization procedure for a multivariable geometric mean [1], we propose a definition for a weighted geometric mean of n positive definite matrices, where the weights vary over all n-dimensional positive probability vectors. We show that the weighted mean satisfies multidimensional versions of all properties that one would expect for a two-variable weighted geometric mean;

(P1) $\mathfrak{B}_n(\omega;A_1,\ldots,A_n) = A_1^{w_1}\cdots A_n^{w_n}$ for commuting A_i's;

(P2) (Joint homogeneity);

$$\mathfrak{B}_n(\omega;a_1A_1,\ldots,a_nA_n) = a_1^{w_1}\cdots a_n^{w_n}\mathfrak{B}_n(\omega;A_1,\ldots,A_n);$$

(P3) (Permutation invariance)

$$\mathfrak{B}_n(\omega_\sigma;A_{\sigma(1)},\ldots,A_{\sigma(n)}) = \mathfrak{B}_n(\omega;A_1,\ldots,A_n)$$

for any permutation σ, where $\omega_\sigma = (w_{\sigma(1)},\ldots,w_{\sigma(n)})$;

(P4) (Monotonicity) If $B_i \le A_i$ for all $1\le i\le n$, then $\mathfrak{B}_n(\omega;B_1,\ldots,B_n) \le \mathfrak{B}_n(\omega;A_1,\ldots,A_n)$;

(P5) (Continuity) The map $\mathfrak{B}_n(\omega;\cdot)$ is continuous;

(P6) (Congruence invariance)

$$\mathfrak{B}_n(\omega;M^*A_1M,\ldots,M^*A_nM) = M^*\mathfrak{B}_n(\omega;A_1,\ldots,A_n)M;$$

(P7) (Joint concavity) For $0\le t\le 1$,

$$\mathfrak{B}_n(\omega;A_1+(1-t)B_1,\ldots,A_n+(1-t)Bn) \ge t\mathfrak{B}_n(\omega;A_1,\ldots,A_n)+(1-t)\mathfrak{B}_n(\omega;B_1,\ldots,B_n)$$

(P8) (Self-duality);
$\mathfrak{B}_n(\omega;A_1^{-1},\ldots,A_n^{-1})^{-1} = \mathfrak{B}_n(\omega;A_1,\ldots,A_n)$;

(P9) (Determinantal identity)

$$\mathrm{Det}\mathfrak{B}_n(\omega;A_1,\ldots,A_n) = \prod_{i=1}^n(\mathrm{Det}A_i)^{w_i};$$

and

(P10) (AGH mean inequalities)

$$(\sum_{i=1}^n w_iA_i^{-1})^{-1} \le \mathfrak{B}_n(\omega;A_1,\ldots,A_n) \le \sum_{i=1}^n w_iA_i.$$

[1] D. Bini, B. Meini and F. Poloni, An effective matrix geometric mean satisfying the Ando-Li-Mathias properties, Math. Comp. **79** (2010), 437-452.

Joint work with Jimmie Lawson (Louisiana State University) and Hosoo Lee (Kyungpook National University)

Hermitian metrics and matrix means

M. Pálfia, Budapest University of Technology and Economics, Hungary
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Fri 11:25, Auditorium

Recently there has been great interest in extending matrix means to several variables. Many authors considered more or less similar iterative methods to construct a multi-variable form for matrix means as the limit point of these iterative procedures. One of the most widely studied matrix mean is the geometric mean

$$G(A,B) = A^{1/2}(A^{-1/2}BA^{-1/2})^{1/2}A^{1/2}.$$

This mean has several special properties. One of them is that the geometric mean is the midpoint map of the manifold of positive definite matrices endowed with the metric induced by the inner product

$$\langle U,V\rangle_p = Tr\{p^{-1}Up^{-1}V\}$$

defined for the tangent space at p. This space is also a Hermitian symmetric space.

Here we show that every matrix mean is the midpoint map of a hermitian symmetric space defined over the space of positive definite matrices [3]. In particularly we show that this is a special case of a more general phenomenon. Given a holomorphic function that has a unique fixed point and fullfills some other properties, automatically induces a hermitian metric on the space of positive definite matrices. These manifolds also turn out to be Riemannian symmetric spaces so therefore also Lie Groups.

We will show that the geometric mean, the harmonic mean and the arithmetic mean obey this construction, so we get the correct corresponding metrics.

After this we consider an iterative multi-variable extension method for means given as midpoint maps in $k-$convex metric spaces [2]. We use $k-$convexity to show that the procedure converges and we also give bounds on the rate of convergence. Later we consider the center of mass on these spaces and we give upper bounds on the distance of the center of mass of the starting points and the limit point of the iterative procedure. We will also give sufficient conditions for the two points to be identical.

Considering once again the $k-$convexity condition we leave this general setting and move back to the case of hermitian metrics on the space of positive definite matrices. As a conclusion we use this machinery given for $k-$convex metric spaces on these symmetric spaces to extend two-variable matrix means to several variables similarly as in [1].

[1] M. Pálfia, Iterative multi-variable extensions to the two-variable mean of positive-definite matrices, SIAM J. Matrix Anal. Appl., to appear.

[2] M. Pálfia, Midpoint maps in metric spaces and the center of mass, preprint.
[3] M. Pálfia, Hermitian symmetric spaces and means of positive definite matrices, in preparation.

Pólya-Szegö inequality for the chaotically geometric mean

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Wed 11:00, Auditorium

Greub-Rheinboldt showed the generalized Pólya-Szegö inequality, which is equivalent to the Kantorovich inequality: Let A and B be commuting positive operators on a Hilbert space H such that $mI \le A, B \le MI$ for some scalars $0 < m < M$. Then $\sqrt{(Ax,x)(Bx,x)} \le \frac{M+m}{2\sqrt{Mm}}(A^{\frac{1}{2}}B^{\frac{1}{2}}x,x)$ for every unit vector $x \in H$. Fujii, Izumino, Nakamoto and Seo showed the non-commutative version: Let A and B be positive operators on H such that $mI \le A, B \le MI$ for some scalars $0 < m < M$. Then $\sqrt{(Ax,x)(Bx,x)} \le \frac{M+m}{2\sqrt{Mm}}(A\sharp Bx,x)$ for every unit vector $x \in H$, where the geometric mean $A\sharp B$ of A and B in the sense of Kubo-Ando is defined by $A\sharp B = A^{1/2}\left(A^{-1/2}BA^{-1/2}\right)^{1/2}A^{1/2}$. Ando-Li-Mathias defined the geometric mean of n-operators and by using it Yamazaki showed an n-variable version of Pólya-Szegö inequality. Moreover, Lawson-Lim defined the weighted geometric mean of n-operators, which extends to the Ando-Li-Mathias geometric mean.

Let $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ be a weight vector if $\sum_{i=1}^n \alpha_i = 1$ and $\alpha_i \ge 0$ for all $i = 1, \cdots, n$. For positive invertible operators $A_1, A_2, \cdots A_n$ on a Hilbert space H, the chaotically geometric mean of $A_1, A_2, \cdots A_n$ for a weight vector α is defined by $\diamondsuit_\alpha(A_1, \cdots, A_n) = \exp\left(\sum_{i=1}^n \alpha_i \log A_i\right)$. If $A_1, A_2, \cdots A_n$ mutually commute, then $\diamondsuit_\alpha(A_1, \cdots, A_n) = A_1^{\alpha_1}\cdots A_n^{\alpha_n}$. The geometric mean $\sharp$ have a monotone property and the chaotically geometric mean does not have a monotone property.

In this talk, we show the chaotically geometric mean version of Pólya-Szegö inequality: Let $A_1, A_2, \cdots A_n$ be positive invertible operators on a Hilbert space H such that $mI \le A_i \le MI$ for some scalars $0 < m < M$ and $i = 1, \cdots, n$. Put $h = \frac{M}{m}$. Then for each weight vector α

$$\frac{1}{S(h)}(\diamondsuit_\alpha(A_1, \cdots, A_n)x, x) \le (A_1x, x)^{\alpha_1}\cdots(A_nx, x)^{\alpha_n} \le S(h)(\diamondsuit_\alpha(A_1, \cdots, A_n)x, x)$$

for every unit vector $x \in H$, where the Specht ratio $S(h)$ is defined by $S(h) = \frac{(h-1)h^{\frac{1}{h-1}}}{e\log h} \quad (h \ne 1, h > 0)$ and $S(1) = 1$.

Operator Monotone Functions, Positive Definite Kernels and Majorization

Mitsuru Uchiyama, Shimane University , Japan
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Wed 12:40, Auditorium

Let $f(t)$ be a real continuous function on an interval, and consider the operator function $f(X)$ defined for Hermitian operators X. We will show that if $f(X)$ is increasing w.r.t. the operator order, then for $F(t) = \int f(t)dt$ the operator function $F(X)$ is convex. Let $h(t)$ and $g(t)$ be C^1 functions defined on an interval I. Suppose $h(t)$ is non-decreasing and $g(t)$ is increasing. Then we will define the continuous kernel function $K_{h,\,g}$ by $K_{h,\,g}(t,s) = (h(t) - h(s))/(g(t) - g(s))$, which is a generalization of the Löwner kernel function. We will see that it is positive definite if and only if $h(A) \le h(B)$ whenever $g(A) \le g(B)$ for Hermitian operators A, B, and give a method to construct a lot of infinitely divisible kernel functions.

[1] M. Uchiyama, Operator Monotone Functions, Positive Definite Kernels and Majorization, to appear PAMS
[2] M. Uchiyama, A new majorization between functions, polynomials, and operator inequalities II, J. Math. Soc. Japan 60(2008) no. 1, 291–310

On properties of geometric mean of n-operators via Riemannian metric

Takeaki Yamazaki, Kanagawa University, Japan
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Tue 17:10, Auditorium

For positive matrices $A_1, \cdots, A_n$, arithmetic mean $\mathfrak{A}(A_1, \cdots, A_n) = \frac{A_1+\cdots+A_n}{n}$ of $A_1, \cdots, A_n$ can be defined by

$$\mathfrak{A}(A_1, \cdots, A_n) = \text{arcmin}\sum_{i=1}^n \|A_i - X\|^2,$$

where arcmin$f(X)$ means the point X_0 at which the function $f(X)$ attains its minimum value and $\|\cdot\|$ means operator norm. If we use Riemannian metric in the above definition instead of operator norm, geometric mean $\mathfrak{G}_\delta(A_1, \cdots, A_n)$ can be considered as

$$\mathfrak{G}_\delta(A_1, \cdots, A_n) = \text{arcmin}\sum_{i=1}^n \delta_2^2(A_i, X).$$

In this talk, we shall introduce properties of geometric mean of n-operators from the view point of operator inequality.

Contributed Talks

A Representation of the Inverse of Tridiagonal Matrices with Nested Functions

J. C. Abderramán Marrero, ETSIT-UPM Technical Univ. of Madrid, Spain.
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Mon 15:00, Room A

The elements of the inverse of any finite tridiagonal matrix can be expressed in terms of determinants of certain tridiagonal submatrices. This gives simple proofs to known properties, [4], and linear recurrences, [2, 3], of the elements of the inverse of a tridiagonal matrix. A direct representation for the elements of the inverse matrix is also achieved by expressing those determinants in terms of nested functions of the elements of the tridiagonal matrix. This is equivalent to the expressions given in [3]. As an illustration, the resolvent matrix, which arises in spectral theory of finite Jacobi matrices, [1], is detailed.

[1] P. C. Gibson, Inverse spectral theory of finite Jacobi matrices, Trans. Amer. Math. Soc. 354 (2002) 4703-4749.
[2] R.K. Kittappa, A representation of the solution of the n-th order linear difference equation with variable coefficients, Linear Algebra Appl. 193 (1993) 211-222.
[3] R.K. Mallik, The inverse of a tridiagonal matrix, Linear Algebra Appl. 325 (2001) 109-139.

[4] G. Strang, T. Nguyen, The interplay of ranks of submatrices, SIAM Review 46:4 (2004) 637-646.

Joint work with M. Rachidi (Académie de Reims, France)

Which digraphs with ring structure are essentially cyclic?

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Fri 15:25, Room C

The Laplacian matrix of a digraph G with vertex set $V(G) = \{1, \ldots, n\}$ and arc set $E(G)$ is the matrix $L = (\ell_{ij}) \in \mathbb{R}^{n\times n}$ in which, for $j \neq i$, $\ell_{ij} = -1$ whenever $(i,j) \in E(G)$, otherwise $\ell_{ij} = 0$; $\ell_{ii} = -\sum_{j\neq i} \ell_{ij},\ i,j \in V(G)$.

We say that a digraph is *essentially cyclic* if its Laplacian spectrum is not completely real. The problem of characterizing essentially cyclic digraphs is difficult and yet unsolved. In the present paper, this problem is solved with respect to the class of *digraphs with ring structure*. By such a digraph we mean a digraph that contains a Hamiltonian cycle and whose remaining arcs belong to the inverse Hamiltonian cycle. Two partial results are as follows:

Theorem 1. *Let L_n be the Laplacian matrix of the digraph G_n whose arcs form the Hamiltonian cycle $(1,n), (n,n-1), \ldots, (2,1)$, the path $(1,2), (2,3), \ldots, (i-1,i)$, and the path $(i+1,i+2), \ldots, (n-1,n)$, where $1 \le i < n$. Then:*

1. *The characteristic polynomial of L_n is $\Delta_{L_n}(\lambda) = Z_i(\lambda)Z_{n-i}(\lambda) - (-1)^n$, where $Z_i(x) = (x-2)Z_{i-1}(x) - Z_{i-2}(x)$, $Z_0(x) \equiv 1$, and $Z_1(x) \equiv x - 1$.*
2. *If n is even, then G_n is essentially cyclic for all i except for $i = \frac{n}{2}$, in which case the eigenvalues of L_n are $4\cos^2 \frac{\pi k}{n}$ and $4\cos^2 \frac{\pi k}{n+2}$, $k = 1, \ldots, \frac{n}{2}$.*
3. *If n is odd, then G_n is essentially cyclic for all i except for $i = \frac{n-1}{2}$ and $i = \frac{n+1}{2}$, in which case the eigenvalues of L_n are $4\cos^2 \frac{\pi k}{n+1}$, $k = 1, \ldots, n$.*

Theorem 2. *Let G_n be a digraph on $n > 3$ vertices constituted by the cycle $(1,n), (n,n-1), \ldots, (2,1)$ and the opposite cycle $(1,2), (2,3), \ldots, (n-1,n), (n,1)$ in which i $(2 < i < n)$ arcs are missing. Then G_n is essentially cyclic and the Laplacian characteristic polynomial of G_n is $\Delta_{L_n}(\lambda) = \prod_{k=1}^{K} Z_{i_k}(\lambda) - (-1)^n$, where $i_1, \ldots, i_K$ are the path lengths in the decomposition of $(1,n), (n,n-1), \ldots, (2,1)$ into the paths linking the consecutive vertices of indegree 1 in G_n.*

The polynomials $Z_i(x)$ are closely related to the Chebyshev polynomials.

We also consider the problem of essential cyclicity for weighted digraphs.

Joint work with P. Chebotarev (Institute of Control Sciences of RAS)

Investigating the Numerical Range and q-Numerical Range of Non Square Matrices

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Thu 15:00, Room A

Let $\mathcal{M}_{m,n}(\mathbb{C})$ be the algebra of $m \times n$ complex matrices. For $m = n$, $F(A) = \{\langle Ax, x\rangle : x \in \mathbb{C}^n, \|x\|_2 = 1\}$ is the *numerical range* of A [3]. Recently, it has been proposed [2] as numerical range of $A \in \mathcal{M}_{m,n}$ *with respect to* $B \in \mathcal{M}_{m,n}$ the compact and convex set

$$w_{\|\cdot\|}(A,B) = \bigcap_{z_0\in\mathbb{C}} \mathcal{D}(z_0, \|A - z_0B\|). \qquad (1)$$

Elaborating the eq.(1), we have noticed that $\bigcup_{\|B\|_F\ge 1} w_{\|\cdot\|_F}(A,B) = \mathcal{D}(0, \|A\|_F)$, thus meaning the independence of $w_{\|\cdot\|_F}(A,B)$ by the matrix B, for $\|B\|_F \ge 1$. Another proposal is the notion of the orthogonal projection onto the lower or higher dimensional subspace and we define *with respect to* an $m \times n$ isometry matrix H $(m \ge n)$: $w_l(A) = F(H^*A)$ *or* $w_h(A) = F(AH^*)$.

In this case, we may have $w(A) = \bigcup_H w_l(A) = \bigcup_H w_h(A)$ and even involving (1), we conclude: $w_l(A) \subseteq w_{\|\cdot\|_2}(A,H) \subseteq w_h(A)$.
Further, we generalize the definition of the numerical range in [1] to the *q-numerical range* of $A \in \mathcal{M}_n$ for $q \in [0,1]$ and we prove for any matrix norm

$$F_q(A) = \bigcap_{z_0\in\mathbb{C}} \mathcal{D}(qz_0, \|A - z_0I_n\|).$$

Hence, we may define the *q-numerical range of* $A \in \mathcal{M}_{m,n}$ *with respect to* $B \in \mathcal{M}_{m,n}$ the set

$$w_{\|\cdot\|}(A,B;q) = \bigcap_{z_0\in\mathbb{C}} \{z \in \mathbb{C} : |z - qz_0| \le \|A - z_0B\|,\ \|B\| \ge q,\ q \in [0,1]\}. \qquad (2)$$

Clearly, (2) is a compact and convex set and $w_{\|\cdot\|}(A,B;1) \equiv w_{\|\cdot\|}(A,B)$ in (1).

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[2] Ch. Chorianopoulos, S. Karanasios and P. Psarrakos, A definition of numerical range of rectangular matrices, Linear Multil. Algebra, **57**, 459-475, 2009.
[3] R.A. Horn and C.R. Johnson, Topics in Matrix Analysis, Cambridge University Press, Cambridge, 1991.

Joint work with J. Maroulas (National Technical University of Athens)

Structured matrix algorithms for solving the Marchenko integral equations

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Thu 15:00, Room C

The initial-value problem for the focusing nonlinear Schroedinger (NLS) equation

$$\begin{cases} \mathbf{i}\, q_t = q_{xx} + 2q|q|^2, & x \in R,\ t > 0, \\ q(x;0) \quad \text{i.c.}, & x \in R, \end{cases}$$

can be solved by following the various steps of the Inverse Scattering Transform (IST) [1]. Among them, a crucial step consists of the numerical solution of two coupled systems of Marchenko integral equations whose kernels are structured. In fact, their solution uniquely specifies the potential $q(x,t)$ and its energy density at each point $x \in R$ and $t \ge 0$.

We illustrate numerical algorithms for solving the Marchenko systems that take advantage of the Hankel structure of the kernels.

[1] C. van der Mee, Direct and inverse scattering for skew-selfadjoint Hamiltonian systems. In: J.A. Ball, J.W. Helton, M. Klaus, and L. Rodman (eds.), *Current Trends in Operator Theory and its Applications*, Birkhäuser OT **149**, Basel and Boston, 2004, pp. 407–439.

Joint work with S. Seatzu, C. van der Mee, G. Rodriguez (University of Cagliari)

Multivariate and directional majorization on $\mathbf{M}_{n,m}$

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Thu 17:35, Room B

Let $\mathbf{M}_{n,m}$ be the set of all $n \times m$ matrices with entries in $\mathbb{R}$. A square matrix D is called doubly stochastic if it has nonnegative entries and $De = e = D^t e$, where $e = (1, 1, ..., 1)^t$. For $A, B \in \mathbf{M}_{n,m}$, it is said that B is multivariate majorized by A if there exists an $n \times n$ doubly stochastic matrix D such that $B = DA$ and it is said that B is directionally majorized by A if for every $x \in \mathbb{R}^n$ there exists an $n \times n$ doubly stochastic matrix D_x such that $Bx = (D_x)Ax$. It is clear that the multivariate majorization implies the directional majorization but the converse is not true. In this paper we investigate some cases where the multivariate and directional majorization are equivalent on $\mathbf{M}_{n,m}$.

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[2] A.W. Marshall, I. Olkin, Inequalities: Theory of Majorization and its Applications, Academic Press , New York, 1979.
[3] F. Martinez Peria, P. Massey and L. Silvestre, Weak matrix majorization, *Linear Algebra Appl.* **403** (2005) 343-368.

Second order pseudospectra of normal matrices

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Thu 11:00, Room A

Let $A \in \mathbb{C}^{n\times n}$ be a normal matrix; a well-known theorem asserts that for all $\varepsilon \geq 0$ the ordinary ε-pseudospectrum of A, $\Lambda_\varepsilon(A)$, is the union of the closed disks of radius ε centered at the eigenvalues of A. We will give a proof of the converse theorem.

Let us define the second order ε-pseudospectrum of any matrix $M \in \mathbb{C}^{n\times n}$ as the set of complex numbers z such that there exists a $\Delta \in \mathbb{C}^{n\times n}$ which satisfies $\|\Delta\| \leq \varepsilon$ and z is a multiple eigenvalue of $M + \Delta$. Let us denote this set by $\Lambda_{\varepsilon,2}(M)$. Here $\|\cdot\|$ stands for the spectral norm.

In this talk we will present a proof of the fact that for any normal matrix A, the set $\Lambda_{\varepsilon,2}(A)$ is a union of closed disks, whose centers and radiuses will be determined in terms of the eigenvalues of A and ε.

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[2] A.N. Malyshev. A formula for the 2-norm distance from a matrix to the set of matrices with multiple eigenvalues. *Numer. Math.* **83** (3), pp. 443-454, 1999.

Joint work with Juan-Miguel Gracia (The University of the Basque Country, Spain) and Francisco E. Velasco (The University of the Basque Country, Spain)

Spectral regularity of Banach algebras of operators

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Fri 16:45, Room Galilei

Let $\mathcal{B}$ be a Banach algebra with unit element. If D is a bounded Cauchy domain in the complex plane and f is an analytic $\mathcal{B}$-valued function taking invertible values on the boundary ∂D of D, the contour integral

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda)f(\lambda)^{-1}d\lambda \qquad (1)$$

is well-defined. By Cauchy's theorem, it is equal to the zero element in $\mathcal{B}$ when f has invertible values on all of D. The Banach algebra $\mathcal{B}$ is said to be *spectrally regular* if the converse of this is true. This means that (1) can only vanish in the trivial case where f takes invertible values on all of D. If $\mathcal{B} = \mathbb{C}$, the integral (1) counts the number of zeros of f inside D; hence $\mathbb{C}$ is spectrally regular. More generally, as a straightforward consequence of a result by A.S. Markus and E.I. Sigal (1970), this also holds for the matrix algebra $\mathbb{C}^{n\times n}$. The Banach algebra $\mathcal{L}(X)$ of all bounded linear operators on an infinite dimensional Banach space X is generally not spectrally regular (example: $X = \ell_2$). In the talk we discuss sufficient conditions for spectral regularity of the Banach subalgebra $\mathcal{L}(X;\mathcal{M})$ of $\mathcal{L}(X)$ consisting of the bounded linear operators on X leaving invariant all members of a given collection $\mathcal{M}$ of closed subspaces of X. New aspects of noncommutative Gelfand theory play a central role.

Joint work with T. Ehrhardt (Santa Cruz, California) and B. Silbermann (Chemnitz, Germany)

On the boundary of the Krein space tracial numerical range

Natalia Bebiano, University of Coimbra, Portugal

Thu 15:25, Room A

Let J be a Hermitian involutive $n \times n$ complex matrix with signature $(r, n-r)$, $0 \leq r \leq n$. We consider $\mathbb{C}^n$ endowed with the indefinite inner product defined by $[x, y] = y^*Jx$, $y, x \in \mathbb{C}$.

For any two $n \times n$ complex matrices C and A, the *J-tracial numerical range* of A (with respect to C), is denoted and defined as:

$$W_C^J(A) = \left\{\mathrm{tr}(CUAU^{-1}) : \quad U \text{ belongs to the } J\text{-unitary group}\right\}.$$

This set is connected in the Gaussian plane $\mathbb{C}$, it has a symmetry property, namely $W_C^J(A) = W_A^J(C)$, and several convexity results for this set are known.

In this talk, the boundary generating curve of $W_C^J(A)$ are obtained and the connection between the J-normality of A and the smoothness of $W_C^J(A)$ is deduced.

Joint work with H. Nakazato (Hirosaki University), Ana Nata (Polytechnic Institute of Tomar, Portugal), J. P. da Providência (University of Coimbra, Portugal)

Computing the block factorization of complex Hankel matrices: application to the Euclidean algorithm

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Fri 15:25, Room B

In this work, we present an algorithm for finding an approximate block diagonalization of complex Hankel matrices via an inversion techniques of an upper triangular Toeplitz matrix, specifically, by simple forward substitution. Our method is based on the results of [1] for computing an approximate block diagonalization of real Hankel matrices. We also consider an approximate block diagonalization of complex Hankel matrices via Schur complementation. An application of our algorithm by calculating the "approximate" polynomial quotient and remainder appearing in the Euclidean algorithm is also given. We have implemented our algorithms in Matlab. Numerical examples are included. They show the effectiveness of our strategy.

[1] S. Belhaj, A fast method to block-diagonalize a Hankel matrix, Numer. Algor., 47, pp. 15–34, 2008.

Matrix Polynomials in the Max Algebra; Eigenvalues, Eigenvectors and Inequalities

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Thu 15:25, Room B

The max algebra consists of the set of nonnegative real numbers together with two binary operations: maximization denoted by $\oplus$ and multiplication denoted by $\otimes$. Matrix operations over the max algebra are defined in the natural manner. We consider matrix polynomials of the form

$$P(\lambda) = A_0 \oplus \lambda A_1 \oplus \cdots \oplus \lambda^{m-1} A_{m-1}$$

where $A_0, A_1, \ldots, A_{m-1} \in \mathbb{R}^{nxn}$ are nonnegative matrices. Specifically, in the sprit of [1], we first present a version of the Perron-Frobenius Theorem [2] for polynomials of this type. Applications of this result to the convergence properties of multistep difference equations over the max algebra are also described. Finally, we discuss the relation between $\mu(P(\lambda))$, the largest max eigenvalue of $P(\lambda)$, and the maximal cycle geometric mean, $\mu(P(1))$, of the nonnegative matrix $P(1)$. Several inequalities relating $\mu(P(\lambda))$ and $\mu(P(1))$, echoing similar results for the conventional algebra, are described.

[1] P. J. Psarrakos and M. J. Tsatsomeros, A primer of Perron- Frobenius theory for matrix polynomials, Linear Algebra Appl. 393 (2004) 333-351.
[2] R.B. Bapat, A max version of the PerronFrobenius theorem, Linear Algebra Appl. 275-276 (1998) 3-18.

Joint work with Oliver Mason (Hamilton Institute, National University of Ireland, Maynooth)

The matrix equation $XA - AX = f(X)$

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Thu 15:25, Room C

Let f be an analytic function defined on a complex domain Ω and $A \in \mathcal{M}_n(\mathbb{C})$. We assume that there exists unique α satisfying $f(\alpha) = 0$. When $f'(\alpha) = 0$ and A is nonderogatory, we solve completely the equation $XA - AX = f(X)$. This generalizes Burde's result. When $f'(\alpha) \neq 0$, we give a method to solve completely the equation $XA - AX = f(X)$: we reduce the problem to solve a sequence of Sylvester equations. Solutions of the equation $f(XA - AX) = X$ are also given in particular cases.

The importance of a dummy paper

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Wed 11:50, Room A

Link analysis has been proposed recently as a tool for ranking scientific publications. For example, in [1,2] a collection of papers is modeled as the states of a Markov chain, with a transition probability associated with every citation. To enforce regularity, the chain is modified by adding a state associated to a *dummy paper*, which cites and is cited by all the papers in the collection. Paper ranking is obtained by computing the invariant probability vector of the modified chain. Not very surprisingly, in this model the dummy paper receives the highest score.

In similar contexts, as the Google search engine or the EigenFactor bibliometric index, regularity of the Markov chain is obtained by allowing random jumps from every state to every other state, performed with a prescribed probability usually tuned by means of a parameter $0 \leq \alpha < 1$.

In this talk we show that the two approaches give rise to two out of a wider family of models, depending on n parameters $0 \leq \alpha_i < 1$, where $i = 1, \ldots, n$ and n is the number of states. The parameter α_i tunes the probability of the random jump from state i or, equivalently, the probability of the transition from the i-th paper to the dummy paper. These parameters can be used to introduce time dependent features in the models e.g., by lowering parameter values of older states. Within this family of models, we study the problem of node updating, which for a generic Markov chain is quite difficult. We show that a certain subfamily, which includes the dummy paper model, has desirable properties from this point of view, generalizing a result presented in [1].

[1] D. A. Bini, G. Del Corso, F. Romani. A combined approach for evaluating papers authors and scientific journals, *Technical Report TR-08-10, Dipartmento di Informatica, University of Pisa*, 2008.
[2] D. A. Bini, G. Del Corso, F. Romani. Evaluating scientific products by means of citation-based models: a first analysis and validation *ETNA* 33 (2008-2009), 1–16.

Joint work with D. Fasino (University of Udine)

Algebraic reflexivity for semigroups of operators

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Tue 11:00, Room B

Let V be a vector space over a field $\mathbb{F}$. For a non-empty set $\mathcal{T}$ of linear transformations on V, let $\operatorname{Lst}\mathcal{T}$ be the family of all $\mathcal{T}$-invariant subsets of V. For a non-empty family $\mathfrak{M}$ of subsets of V, let $\operatorname{Sgr}\mathfrak{M}$ be the set of all linear transformations T on V satisfying $\mathfrak{M} \subset \operatorname{Lst} T$. Then $\operatorname{Lst}\mathcal{T}$ is a lattice with respect to the taking unions and intersections. $\operatorname{Sgr}\mathfrak{M}$ is a multiplicative semigroup of linear transformations. It is easily seen that $\mathcal{T} \subseteq \operatorname{Sgr}\operatorname{Lst}\mathcal{T}$. A multiplicative semigroup $\mathcal{S}$ of linear transformations is said to be algebraicaly reflexive if $\operatorname{Sgr}\operatorname{Lst}\mathcal{S} = \mathcal{S}$.

We study algebraic reflexivity of multiplicative semigroups of linear transformations and give some examples of algebraic reflexive semigroups. At the end we characterize those bounded linear operators on a complex Banach space that are determined by the lattice of invariant subsets.

[1] J. Bračič, Algebraic reflexivity for semigroup of operators, Electron. J. Linear Algebra, 18, pp. 745-760, 2009.

Lorentzian Distance Matrices

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Tue 11:50, Room A

We consider distance matrices in the Lorentzian n-space, $\mathbb{R}^{1,n-1}$. A matrix $D = [d_{ij}]_{i,j=1,\ldots,m}$ is said to be a Lorentzian distance matrix if there exists a set of points of $\mathbb{R}^{1,n-1}$, $\mathcal{X} = \{x_1, x_2, \ldots, x_m\}$, such that $d_{ij} = ||x_i - x_j||_\circ^2$, where $||.||_\circ$ denotes the Lorentzian norm.

In this study we present an alternative proof for a classical characterization, due to [1], of this type of matrices. Other characterizations are also taken into consideration. It is known that every Euclidian distance matrix is an elliptic matrix, we prove that every elliptic matrix is a Lorentzian distance matrix. With this framework, we investigate how to distinguish the elliptic matrices that are strictly Lorentzian (*i.e.*, non Euclidian).

[1] I. J. Shoenberg. Remarks to Maurice Frechet's Article "Sur La Definition Axiomatique D'Une Classe D'Espace Distances Vectoriellement Applicable Sur L'Espace De Hilbert". *The Annals of Mathematics, 2nd Ser.*, Vol. 36, No. 3. (Jul., 1935), pp. 724-732.

Joint work with A. Breda (University of Aveiro)

On the gaps in the set of exponents of primitive boolean circulant matrices

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Thu 17:10, Room C

It is well-known that the maximum exponent that an n-by-n boolean primitive circulant matrix can attain is $n-1$. We consider the problem of describing the possible exponents attained by these kind of matrices. This problem is equivalent to the following two problems: 1) finding the set of exponents attained by primitive Cayley digraphs on a cyclic group ; 2) determining the set of orders of bases for $\mathbb{Z}_n$. We present a conjecture for the possible such exponents and prove this conjecture in several cases. We also find the maximum exponent that n-by-n boolean primitive circulant matrices with constant number of nonzero entries in its generating vector can attain and give matrices attaining such exponents.

Joint work with S. Furtado (Faculdade de Economia do Porto, Portugal)

Naturally graded n-dimensional Leibniz algebras of nilindex $n-3$.

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Fri 17:10, Room Galilei

Leibniz algebras present a "non commutative" analogue of Lie algebras and they were introduced by J.-L. Loday, [5], as algebras which satisfy the following Leibniz identity: $[x,[y,z]] = [[x,y],z] - [[x,z],y]$.

It should be noted that Lie algebras are particular cases of Leibniz algebras. For a given Leibniz algebra L we consider lower central series: $L^1 = L$ and $L^{k+1} = [L^k, L^1], \quad k \geq 1$.

A Leibniz algebra L is called nilpotent if there exists $s \in \mathbb{N}$ such that $L^s = \{0\}$. The minimum number satisfying this property is called the nilindex of L. For an n-dimensional Leibniz algebra, we have the natural filtration:

$$L \supseteq L^2 \supseteq \cdots \supseteq L^{n-3} \supseteq L^{n-2} \supseteq L^{n-1} \supseteq L^n \supseteq L^{n+1} = \{0\}.$$

Then the description of n-dimensional algebras L with the following conditions: $L^{n-i} \neq \{0\}, L^{n-i+1} = \{0\}, 0 \leq i \leq n-1$ for any value of i gives pairwise non isomorphic classes of algebras, more precisely, for different i the defined classes of algebras are disjoint. Evidently, the nilindex of an n-dimensional algebra does not exceed $n+1$. A Leibniz algebra is called zero-filiform, filiform and quasi-filiform, if its nilindex is equal to $n+1$, n and $n-1$, respectively. The classification of naturally graded algebras is obtained already. In other words, n-dimensional naturally graded Leibniz algebras with length of the natural filtration equal to $n+1$, n and $n-1$ are known [1], [2], [3] and [4]. The descriptions of some subclasses of naturally graded Leibniz algebras with length of the filtration $n-2$ were obtained. The main result of this work is to complete the classification of complex n-dimensional naturally graded Leibniz algebras with length of the filtration equal to $n-2$.

[1] Sh.A. Ayupov, B.A. Omirov, On some classes of nilpotent Leibniz algebras, (Russian) Sibirsk. Mat. Zh., 42 (1), pp. 18-29, 2001; translation in Siberian Math. J., 42 (1), pp. 15-24, 2001.
[2] L.M. Camacho, J.R. Gómez, A.J. González, B.A. Omirov, Naturally graded quasi-filiform Leibniz algebras, Journal of Symbolic Computation, 44, pp. 527–539, 2009.
[3] L.M. Camacho, J.R. Gómez, A.J. González, B. A. Omirov, Naturally graded 2-filiform Leibniz algebras, Communications in Algebra, To appear.
[4] J.R. Gómez, A. Jiménez-Merchán, Naturally graded quasi-filiform Lie algebras, J. Algebra, 256(1), pp. 211-228, 2002.
[5] J.L. Loday, Une version non commutative des algébres de Lie: les algébres de Leibniz, Ens. Math., 39, pp. 269-293, 1993.

Joint work with L.M. Camacho (Universidad Sevilla), J.R. Gómez (Universidad Sevilla), Sh.B. Redjepov (Institute of Mathematics and Information Technologues, Uzbekistan Academy of Science)

Block-diagonal stability for switched systems

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Fri 16:45, Room Fermi

A switched linear system is a family of time invariant linear systems, called the system bank, together with a switching law that determines how the time invariant systems commute among themselves. We consider switched systems with a finite system bank $\{\Sigma_p = (A_p, B_p, C_p, D_p) : p \in \mathcal{P}\}$, where $\mathcal{P}$ a finite index set. It is a well-known fact that the existence of a positive definite matrix P such that $A_p^T P + PA_p < 0$, for all $p \in \mathcal{P}$, implies the stability of the overall switched system, under arbitrary switching. In this case, the time invariant system Σ_p are said to have a common quadratic Lyapunov function.

In this work, we assume that the system matrices A_p have a pre-specified block structure and we investigate the existence of a common quadratic Lyapunov function with block-diagonal structure.

[1] Isabel Brás, Ana Carapito, Paula Rocha, Block-diagonal stability for switched systems, *In preparation.*

Joint work with Isabel Brás (Universidade de Aveiro) and Paula Rocha (Universidade do Porto)

The set of feedback assignable polynomials to a non-controllable single-input linear system

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Tue 17:10, Room B

A canonical form for generic single input linear systems over a Bézout domain R (including non-reachable/non-controllable cases) is given. This canonical form can be used to compute effectively the set of assignable polynomials of a given linear system and some feedback invariants.

We also generalize a classical result in control theory by proving that given a Bézout domain and a single input linear system $\Sigma = (A, \underline{b}) \in R^{n\times n} \times R^{n\times 1}$, the smallest principal ideal $(q) \subseteq R[z]$ of $R[z]$ containing $\mathcal{U}_n(z\mathbf{1} - A, \underline{b})$ is a feedback invariant and divides all feedback assignable polynomials to Σ.

(k,τ)-regular sets of circulant graphs

P. Carvalho, University of Aveiro, Portugal
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Fri 15:00, Room C

Given a graph $G = (V(G), E(G))$, a subset of vertices $\emptyset \neq S \subseteq V(G)$ is a (k,τ)-regular set if S induces a k-regular subgraph in G and every vertex in $V(G)\backslash S$ has exactly τ neighbors in S. In this presentation we introduce some results on the characterization of (k,τ)-regular sets for circulant graphs with symbol that fulfills some requirements and we prove the existence of (k,τ)-regular sets for certain values on the order of G, namely, $|V(G)|$ even and $|V(G)|$ multiple of 3. According to [1], a subset $\emptyset \neq S \subseteq V(G)$ of a regular graph is a (k,τ)-regular set if and only if $k-\tau$ is an eigenvalue of G. Since circulant graphs are regular graphs, from the above results we obtain a combinatorial characterization of the spectrum of circulant graphs.

[1] D. M. Thompson, Eigengraphs: constructing strongly regular graphs with block designs, Utilitas Math., 20, pp. 83-115, 1981.

Joint work with P. Rama (University of Aveiro)

Low-Rank Approximation of Graph Similarity Matrices

Thomas P. Cason, Université catholique de Louvain, Belgium
http://www.inma.ucl.ac.be/∼cason/
Mon 11:50, Room A

Graphs are a powerful tool for many practical problems such as pattern recognition, shape analysis, image processing and data mining. Measures of graph similarity have a broad array of applications, including comparing chemical structures, navigating complex networks like the World Wide Web, and analyzing different kinds of biological data [1].

Blondel *et al.* introduced the notion of similarity between nodes of two graphs in [2]. They defined a similarity measure as a fixed point of the even iterates of the following recurrence

$$S_0 = \mathbf{1}_{m,n}\ , \qquad S_{k+1} = \mathcal{M}(S_k)/\left\|\mathcal{M}(S_k)\right\|,$$

where $\mathcal{M}(S) := ASB^T + A^TSB$ and A and B are graph adjacency matrices. One can prove that the similarity matrix is solution of $\max_{\langle S,S\rangle=1} \Phi(S) = \operatorname{tr}\left(S^T\mathcal{M}^2(S)\right)$. When S is large, the iteration becomes computationally expensive. Hence one can think to modify the problem in order to find an approximation of S at lower cost. In this work, we consider the approximation of the similarity matrix S in $\mathcal{S}$, the set of matrices of norm 1 and rank at most k.

We propose the following algorithm to find stationary points of Φ

$$S_+ := \arg\max_{\tilde{S}\in\mathcal{S}} \operatorname{tr}\left(\tilde{S}^T\mathcal{M}^2(S)\right) \tag{1}$$

The maximum is achieved when S is aligned with the dominant space of $\mathcal{M}^2(S)$. One iteration of (1) costs

$$6\ (m^2+n^2)\ k + 17\ (m+n)\ k^2 + O(k^3)$$

whereas one full rank iteration costs $4(m^2n + n^2m)$.

We characterize the fixed points of (1) and prove that all accumulation points are stationary points of $\Phi(S)$. Preliminary results were presented in [3].

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Joint work with P.-A. Absil and P. Van Dooren (UC Louvain, Belgium)

Perturbation analysis of Markov chains: a matrix calculus approach

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Wed 11:50, Room B

Whenever Markov chains are used as models of real-world phenomena, perturbation analysis, quantifying the sensitivity of conclusions to changes in parameters, is an important problem. The Magnus-Neudecker formalism for matrix calculus provides easily computable solutions for many such problems. Here, I will summarize some recent results on the perturbation analysis of absorbing and ergodic finite-state Markov chains. In absorbing chains, interest focuses on questions related to the time to absorbtion, and a key to these results is the fundamental matrix $\mathbf{N} = (\mathbf{I}-\mathbf{U})^{-1}$, where $\mathbf{U}$ is the matrix of transition probabilities among the transient states. Suppose that $\mathbf{U}$ is a function of a parameter vector θ. Then it can be shown that

$$\frac{d\mathrm{vec}\mathbf{N}}{d\theta^{\mathsf{T}}} = \left(\mathbf{N}^{\mathsf{T}} \otimes \mathbf{N}\right) \frac{d\mathrm{vec}\mathbf{U}}{d\theta^{\mathsf{T}}} \tag{1}$$

where the vec operator stacks columns of a matrix one above the next, and $\otimes$ denotes the Kronecker product. Extensions of this result will be shown for the sensitivity and elasticity of the moments of the time to absorbtion, for discrete- and continuous-time absorbing chains. When applied to ergodic chains, the approach yields the sensitivity of the stationary distribution $\hat{\mathbf{p}}$. Let $\mathbf{P}$ be the transition matrix, assumed to be a function of a parameter vector θ; then

$$\frac{d\hat{\mathbf{p}}}{d\theta^{\mathsf{T}}} = \left(\mathbf{I}-\mathbf{P}+\hat{\mathbf{p}}\mathbf{e}^{\mathsf{T}}\mathbf{P}\right)^{-1}\left(\hat{\mathbf{p}}^{\mathsf{T}}\otimes\mathbf{I}-\hat{\mathbf{p}}^{\mathsf{T}}\otimes\hat{\mathbf{p}}\mathbf{e}^{\mathsf{T}}\right)\frac{d\mathrm{vec}\mathbf{P}}{d\theta^{\mathsf{T}}} \tag{2}$$

where $\mathbf{e}$ is a vector of ones. I will illustrate the results with some ecological and demographic applications.

Combinatorial Identities from LU Decomposition of Matrices

Marc Chamberland, Grinnell College, USA
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Mon 11:25, Room A

The LU decomposition is a standard tool used in numerical linear algebra. This talk shows how this tool may be used to obtain combinatorial identities, some of which are new. As a simple example, choose the (i,j) entry of a 6×6 matrix to be F_{i+j-1}^2, where F_k is the k^{th} Fibonacci number. An LU decomposition produces

$$\begin{bmatrix} 1 & 1 & 4 & 9 & 25 & 64 \\ 1 & 4 & 9 & 25 & 64 & 169 \\ 4 & 9 & 25 & 64 & 169 & 441 \\ 9 & 25 & 64 & 169 & 441 & 1156 \\ 25 & 64 & 169 & 441 & 1156 & 3025 \\ 64 & 169 & 441 & 1156 & 3025 & 7921 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 4 & 5/3 & 1 & 0 & 0 & 0 \\ 9 & 16/3 & 2 & 1 & 0 & 0 \\ 25 & 13 & 6 & 0 & 1 & 0 \\ 64 & 35 & 15 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 & 9 & 25 & 64 \\ 0 & 3 & 5 & 16 & 39 & 105 \\ 0 & 0 & 2/3 & 4/3 & 4 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Recognizing the terms in the factored matrices yields the identity

$$F_{i+j-1}^2 = F_i^2 F_j^2 + \frac{1}{3}F_{i-1}F_{i+2}F_{j-1}F_{j+2} + \frac{2}{3}F_{i-2}F_{i-1}F_{j-2}F_{j-1}$$

Many diverse identities will be given by performing an LU decomposition on matrices whose terms involve binomial coefficients, number theoretic functions, orthogonal polynomials, q-series, and multiple derivatives of functions.

Graph Laplacians and Logarithmic Forest Distances

P. Chebotarev, Institute of Control Sciences of the RAS, Russia
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Fri 17:35, Room C

A new parametric family of distances for graph vertices is proposed. At the extreme values of the parameter, the family generates the shortest-path distance and the resistance distance (coinciding with the commute time distance). A distinctive feature of the family members is that they are graph-geodetic: $d(i,j) + d(j,k) = d(i,k)$ if and only if every path from i to k passes through j. The family is constructed as follows:

$$Q_\alpha = (I + \alpha L)^{-1},$$

where $\alpha \in \mathbb{R}_+$ is a parameter and L is the Laplacian matrix of the graph,

$$H_\alpha = \gamma\,(\alpha - 1)\,\overrightarrow{\log_\alpha Q_\alpha},$$

where $\alpha \neq 1$, $\gamma \in \mathbb{R}_+$, and the logarithm $\overrightarrow{\log_\alpha Q_\alpha}$ is taken entrywise, and finally,

$$D_\alpha = \tfrac{1}{2}(h_\alpha \mathbf{1}^{\mathrm{T}} + \mathbf{1} h_\alpha^{\mathrm{T}}) - H_\alpha,$$

where h_α is the column of the diagonal entries of H_α and $\mathbf{1} = (1, \ldots, 1)^{\mathrm{T}}$, provides the matrix of distances. The proofs of the properties of the family [1] involve the matrix forest theorem [2] and the graph bottleneck inequality [3]. On the possible applications, see [4]. A sensible choice of the scaling parameter γ is $\gamma = \ln(e + \alpha^{\frac{2}{n}})$. The distances are called the *logarithmic forest distances.*

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Eigenpairs of Adjacency Matrices of Balanced Signed Graphs

Mei-Qin Chen, Department of Mathematics and Computer Science, The Citadel
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Thu 16:45, Room C

In this paper, we present results on eigenvalues λ and their associated eigenvectors x of an adjacency matrix A of a balanced signed graph. A graph $G = (V, E)$ consists of a set V of vertices and a set E of edges between two adjoined vertices. A signed graph is a graph for which each edge is labeled with either $+$ or $-$. A signed graph is said to be balanced if there are an even number of negative signs in each cycle (a simple closed path).

Signed graphs were first introduced and studied by F. Harary to handle a problem in social psychology. It was shown by Harary in 1953 that a signed graph is balanced if and only if its vertex set V can be divided into two sets (either of which may be empty), X and Y, so that each edge between the sets is negative and each within a set is positive. Based on this fundamental theorem for balanced signed graphs, vertices of a balanced signed graph can be labeled in a way so that its adjacency matrix is well structured. Using this special structure, we find exactly all eigenvalues and their associated eigenvectors of the adjacency matrix A of a given balanced signed graph. We will present eigenpairs (λ, x) of adjacency matrices of three types of balanced signed graphs: (1) graphs that are complete; (2) graphs with t vertices in X or in Y that are not connected; and (3) graphs that are bipartite.

Joint work with Spencer P. Hurd (The Citadel)

On classical adjoint-commuting mappings between matrix algebras

Wai-Leong Chooi, University of Malaya, Malaysia.
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Fri 17:35, Room Galilei

Let $\mathbb{F}$ be a field and let m and n be integers with $m, n > 2$. Let $\mathcal{M}_n$ denote the algebra of $n \times n$ matrices over $\mathbb{F}$. In this note, we characterize mappings $\psi : \mathcal{M}_n \to \mathcal{M}_m$ that satisfy one of the following conditions:

[1] $|\mathbb{F}| = 2$ or $|\mathbb{F}| > n$, and $\psi(\mathrm{adj}\,(\mathrm{A} + \alpha \mathrm{B})) = \mathrm{adj}\,(\psi(\mathrm{A}) + \alpha\psi(\mathrm{B}))$ for all $A, B \in \mathcal{M}_n$ and $\alpha \in \mathbb{F}$ with $\psi(I_n) \neq 0$.

[2] ψ is surjective and $\psi(\mathrm{adj}\,(\mathrm{A} - \mathrm{B})) = \mathrm{adj}\,(\psi(\mathrm{A}) - \psi(\mathrm{B}))$ for all $A, B \in \mathcal{M}_n$.

Here, adj A denotes the classical adjoint of the matrix A, and I_n is the identity matrix of order n. We give examples showing the indispensability of the assumption $\psi(I_n) \neq 0$ in our results.

Joint work with Wei-Shean Ng (Universiti Tunku Addul Rahman, Malaysia.)

A numerical range for rectangular matrices and matrix polynomials

Ch. Chorianopoulos, National Technical University of Athens, Greece
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Tue 15:00, Room B

The numerical range of an operator can be written as an (infinite) intersection of closed circular discs. This interesting property was observed by Bonsall and Duncan (1973), and leads (in a natural way) to a definition of numerical range of rectangular complex matrices. The new range is always compact and convex, and satisfies basic properties of the standard numerical range. The proposed definition is also extended to the case of matrix polynomials.

Joint work with P. Psarrakos (National Technical University of Athens)

Solution of Non-Symmetric Algebraic Riccati Equations from Transport Theory
Eric King-wah Chu, Monash University, Melbourne, Australia
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Tue 15:50, Room Fermi

Transport theory [1,3] provides a rich source of mathematical problems. For example, from (i) a differential-integral equation in a two-dimensional model, or (ii) a differential equation in a one-dimensional multi-state model, we shall derive and study the non-symmetric algebraic Riccati equation

$$B^- - XF^- - F^+X + XB^+X = 0,$$

where (i) $F^\pm \equiv I - \hat{s}PD^\pm$, $B^- \equiv (\hat{b}I + \hat{s}P)D^-$ and $B^+ \equiv \hat{b}I + \hat{s}PD^+$ with positive diagonal matrices $D^\pm$, a low-ranked P and positive parameters $\hat{b}$ and $\hat{s}$; or (ii) $F^\pm \equiv (I - F)D^\pm$ and $B^- \equiv BD^-$ with possibly low-ranked matrices F and B. These are generalizations of the one studied by Juang in [2].

We prove the existence of the minimal solution X^* under physically reasonable assumptions, and study its numerical computation by fixed point and Newton iterations. We shall also study several special cases. For example, when (i) $\hat{b} = 0$ and P is low-ranked, then $X^* = \hat{s}UV^\top$ is low-ranked; or (ii) when B and F are low-ranked, then $X^* = T \circ (UV^\top)$ with the low-ranked $UV^\top$. The solution can then be computed using more efficient iterative processes. Numerical examples will be given to illustrate our theoretical results.

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Joint work with J. Juang (National Chiao Tung University), T. Li (Southeast University), and W.-W. Lin (National Chiao Tung University)

On normal Hankel matrices
V. N. Chugunov, Institute of Numerical Mathematics, Russian Academy of Sciences, ul.Gubkina 8, Moscow, 119991 Russia
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Mon 15:25, Room A

The normal Hankel problem is the one of characterizing the matrices that are normal and Hankel at the same time. This problem turned out to be much harder than the normal Toeplitz problem, which the authors solved in early 1990's.

In this talk, we give a sketch of the complete solution of the normal Hankel problem. We present a general approach that allows us to obtain all the classes of normal Hankel matrices as special cases of a unified scheme.

Joint work with Kh. D. Ikramov (Faculty of Computational Mathematics and Cybernetics, Moscow State University, Leninskie gory, Moscow, 119992 Russia)

A Lower Bound for the Distance from a Controllable Switched Linear System to an Uncontrollable One
J. Clotet, Universitat Politècnica de Catalunya, Spain
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Tue 15:25, Room A

We consider the set of controllable switched linear systems (SLS). Since the parameters of a given mathematical model are usually determined only approximately, an uncontrollable system may appear as a controllable one. In other words, in general an uncontrollable system becomes controllable when perturbing. Nevertheless, the converse may also occur if perturbations are big enough. In this work we obtain a lower bound for the distance from a controllable SLS to the nearest SLS which is uncontrollable, thus determining a safety neighbourhood for any controllable SLS.

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Joint work with J. Ferrer, M. D. Magret (Universitat Politècnica de Catalunya)

What's New? Matrix Methods for Extracting Update Summaries
John M. Conroy, IDA Center for Computing Sciences, Bowie, MD, USA
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Wed 11:00, Room A

In this talk we will describe the use of linear algebra to develop algorithms to extract information from text documents. The problem is two-fold: Given an initial cluster of documents returned from a query, construct a brief summary of the cluster. Later, given a second cluster of documents relevant to the query, generate an *update* summary, which focuses on what is new. See [1] and [2] for more details.

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[2] NIST. Text analysis conference, http://www.nist.gov/tac, 2009.

Joint work with Judith D. Schlesinger, IDA/CCS & Dianne P. O'Leary, UMCP.

On the faces of faces of the tridiagonal Birkhoff polytope

Liliana Costa, University of Aveiro, Portugal
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Thu 16:45, Room B

Doubly stochastic matrices (*i.e.* real square matrices with nonnegative entries and all rows and columns sums equal to one) have been studied quite extensively. This denomination is associated to probability distributions and it is amazing the diversity of branches of mathematics in which doubly stochastic matrices arise: geometry, combinatorics, optimization theory, graph theory and statistics. In 1946, Birkhoff published a remarkable result asserting that a matrix in the polytope of $n \times n$ nonnegative doubly stochastic matrices, Ω_n, is a vertex if and only if it is a permutation matrix. In fact, Ω_n is the convex hull of all permutation matrices of order n. The *Birkhoff polytope* Ω_n is also known as *transportation polytope* or *doubly stochastic matrices polytope.*

In 2004, Dahl, [3], discussed the subclass of Ω_n consisting of the tridiagonal doubly stochastic matrices and the corresponding subpolytope

$$\Omega_n^t = \{A \in \Omega_n : A \text{ is tridiagonal}\},$$

the so-called *tridiagonal Birkhoff polytope*, and studied the facial structure of Ω_n^t.

In this talk we present an interpretation of p-faces, $p = 0, 1, \ldots$, of the tridiagonal Birkhoff polytope, Ω_n^t, in terms of graph theory. And, for a given p-face of Ω_n^t, we determine the number of faces of dimension zero, one, two or three, that are contained in it and we discuss their nature. In fact, a 2-face of Ω_n^t is a triangle or a quadrilateral and the 3-faces can be at most hexahedrons.

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Joint work with Enide Andrade Martins (University of Aveiro)

Matrices with Prescribed Characteristic Polynomials and Prescribed Entries

G. Cravo, University of Madeira and CELC, Portugal
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Fri 17:10, Auditorium

An important problem that has been studied for some decades, is the description of the possible eigenvalues of a square matrix over a field, when some of its entries are prescribed and the other entries are unknown.

Another important problem that motivates our work is the description of the possible eigenvalues or the characteristic polynomial of a partitioned matrix of the form $A = [A_{i,j}]$, over a field, where the blocks $A_{i,j}$ are of type $p_i \times p_j$ $(i, j \in \{1, 2\})$, when some of the blocks $A_{i,j}$ are prescribed and the others are unknown.

In our work we intend to unify the previous problems. Indeed, our main goal is to describe the possible eigenvalues or the characteristic polynomial of a partitioned matrix of the form $C = [C_{i,j}] \in F^{n \times n}$, where F is an arbitrary field, $n = p_1 + \cdots + p_k$, the blocks $C_{i,j}$ are of type $p_i \times p_j$ $(i, j \in \{1, \ldots, k\})$, and some of its blocks are prescribed and the others vary. For this more general question we just obtained some partial results. In order to give more insight into this problem, we considered the particular situation $k = 3$.

Furthermore, we still analyze the possibility of the pair of the form

$$(C_1, C_2) = \left(\begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,1} \end{bmatrix}, \begin{bmatrix} C_{1,3} \\ C_{2,3} \end{bmatrix} \right)$$

being completely controllable (where the blocks $C_{i,j}$ are of type $p_i \times p_j$, $i \in \{1, 2\}, j \in \{1, 2, 3\}$), when three of its blocks are prescribed.

Pairs of matrices that preserve the value of a generalized matrix function on the set of the upper triangular matrices

Henrique F. da Cruz, Universidade da Beira Interior, Portugal
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Tue 15:00, Room Fermi

Let H be a subgroup of the symmetric group of degree m, let χ be an irreducible character of H and let $\mathbb{F}$ be an arbitrary field of characteristic zero. In this talk we give conditions that characterize the pairs of m-square matrices over $\mathbb{F}$, that leave invariant the value of a generalized matrix function associated with H and χ on the set of the upper triangular matrices, that is, denoting by d_χ^H the generalized matrix function associated with H and χ and by $T_n^U(\mathbb{F})$ the set of m-square upper triangular matrices, we describe the pairs (A, B) of $m \times m$ matrices over $\mathbb{F}$ that satisfies

$$d_\chi^H(AXB) = d_\chi^H(X),$$

for all $X \in T_n^U(\mathbb{F})$.

[1] Rosário Fernandes, Henrique F. da Cruz, Pairs of matrices that preserve the value of a generalized matrix function on the set of the upper triangular matrices, *submitted.*

Joint work with Rosário Fernandes (Universidade Nova de Lisboa)

An algebraic method for solving some evolution problems

Z. Dahmani, University of Mostaganem, Algeria
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Tue 15:50, Room A

In this talk, we employ an algebraic method, which is based on resolution of linear algebraic systems, to derive traveling wave solutions for some nonlinear evolution problems. The obtained solutions include also kink solutions. Using this linear method, we present some examples which appear in various areas of applied mathematics such as modeling of fluid dynamics and population dynamics.

Imputing Missing Entries in a Data Matrix

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Wed 11:25, Room A

The problem of imputing missing entries of a data matrix is easy to state: Some entries of the matrix are unknown and we want to assign "appropriate values" to these entries. The need for solving such problems arises in several applications, ranging from traditional fields to modern ones. Typical traditional fields are Statistical analysis of incomplete survey data, Business Reports, Meteorology and Hydrology. Modern applications arise in Machine Learning, Data Mining, DNA microarrays data, Computer Vision, Recommender Systems and Collaborative Filtering. The problem is highly interesting and challenging. Many ingenious algorithms have been proposed, and there is vast literature on imputing techniques. Yet, most of the papers consider the imputing problem within the context of a specific application. The current survey attempts to provide a broader view of the problem, one that exposes the large variety of existing methods, with focus on linear algebra and optimization issues. Old and new methods are examined and explained. The equivalence theorems that we prove reveal surprising relations between apparently different methods.

The first part of the talk introduces the problem and surveys the main solution approaches. Starting from simple averaging methods we outline some basic imputing algorithms, including iterative column regression (ICR), k nearest neighbors (KNN) imputing and iterative SVD imputing. Then we move on to consider recently proposed methods, such as tail minimization (FRAA), rank minimization, and nuclear norm minimization. As our survey shows, the construction of a low-rank approximating matrix is the ultimate goal of several imputing methods. The second part of the talk considers direct minimization methods that achieve this task. The methods discussed include successive rank-one modifications (SROM), alternating least squares (ALS), Newton, Gauss-Newton, and Wiberg's algorithm.

Obtaining canonical forms associated with the problem of perturbation of one column of a controllable pair

I. de Hoyos, Universidad del País Vasco, Spain
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Fri 17:10, Room Fermi

Let (A, B) be a completely controllable matrix pair. When we consider the problem of characterizing the controllability indices of all the matrix pairs obtained by small perturbations on one column of B, a new equivalence relation arises in a natural way.

This equivalence relation is a kind of partial feedback equivalence. As a consequence the controllability indices are invariant, but they do not form a complete system of invariants.

We have found some new invariants for this equivalence relation. These invariants are of two types: continuous and discrete.

Finally, we have achieved a procedure which allows us to obtain canonical forms in terms of the invariants.

Joint work with I. Baragaña (Universidad del País Vasco), M. A. Beitia (Universidad del País Vasco)

Eigenvalues computation of possibly unsymmetric quasiseparable matrices by LR steps

Gianna M. Del Corso, Department of Computer Science, University of Pisa, Italy
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Fri 12:15, Room Pacinotti

In the last few years many numerical techniques for computing eigenvalues of structured rank matrices have been proposed. Most of them are based on QR iterations since, in the symmetric case, the rank structure is preserved and high accuracy is guaranteed. In the unsymmetric case, however the QR algorithm destroys the rank structure, which is instead preserved if LR iterations are used. We show that almost all quasiseparable matrices can be represented in terms of the parameters involved in their Neville factorization, and that this representation is preserved under LR steps. Moreover, we propose an implicit shifted LR method with a linear cost per step. We show that for totally nonnegative matrices the algorithm is stable and does not incur in breakdown also if the Laguerre shift is used. Computational evidence shows that good accuracy is obtained also when applied to symmetric positive definite matrices.

Joint work with Roberto Bevilacqua (University of Pisa) and Enrico Bozzo (University of Udine)

Preserving quasi-commutativity on self-adjoint operators

G. Dolinar, University of Ljubljana, Slovenia
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Mon 11:00, Room B

Let H be a separable Hilbert space and $\mathcal{B}_{sa}(H)$ the set of all bounded linear self-adjoint operators. We say that $A, B \in \mathcal{B}_{sa}(H)$ quasi-commute if there exists a nonzero $\xi \in \mathbb{C}$ such that $AB = \xi BA$, and that $\Phi\colon \mathcal{B}_{sa}(H) \to \mathcal{B}_{sa}(H)$ preserves quasi-commutativity in both directions when the following holds: $\Phi(A)$ quasi-commutes with $\Phi(B)$ if and only if A quasi-commutes with B. Classification of bijective maps on $\mathcal{B}_{sa}(H)$ which preserve quasi-commutativity in both directions will be presented.

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Joint work with B. Kuzma (University of Primorska)

P-rank corrections for box-constrained global optimization problems

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Fri 15:25, Room Galilei

In previous papers the author et alii showed that BFSG-type methods approximating the hessian of twice continuously differentiable functions with a structured matrix are very efficient to compute local minima, particularly in the secant case. Moreover, by utilizing a suitable BFGS-type algorithm, a general theorem ensuring the convergence to the global minimum of unconstrained twice continuously differentiable functions was recently proved.
A family of important deterministic methods for global optimization is based upon the theory of terminal attractors and repellers. Unfortunately, the utilization of scalar repellers is unsuitable when the dimension n of the problem assumes values of operational interest.
On the other hand, the algorithms founded on the classical

αBB technique are often ineffective for computational reasons, even if, more recently, the utilization of a new class of convex under-estimators and relaxations has significantly improved the performances of this approach.
In order to increase the power of the repeller in the tunneling phase, the utilization of repeller matrices with a proper structure is certainly promising and deserves investigation. More precisely, it is interesting to test the performances obtained by approximating the optimal (unknown) repeller matrix with the sum of a diagonal matrix and a low rank one. The corresponding tunneling phase must be, in fact, properly superimposed in the global optimization algorithm in the frame of the αBB computational scheme.
Numerical experiences on a wide set of classical and well known optimization problems show that the latter approach has a significant effect on the efficiency of the whole global optimization procedure.

Commutativity preservers on matrix algebras
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Tue 12:15, Room B

Let $M_n(\mathbb{F})$ be the algebra of all $n \times n$ matrices over the field $\mathbb{F}$. A map $\phi : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ preserves commutativity if $\phi(A)\phi(B) = \phi(B)\phi(A)$ whenever $AB = BA$, $A, B \in M_n(\mathbb{F})$. If ϕ is bijective and both ϕ and ϕ^{-1} preserve commutativity, then we say that ϕ preserves commutativity in both directions. We will represent recent results on general (non-linear) maps on some matrix algebras that preserve commutativity in both directions or in one direction only. We will talk about complex and real matrices, hermitian, symmetric, and alternate matrices.

On J-normal matrices with J'-normal principal submatrices
S. Furtado, University of Porto and CELC, Portugal
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Tue 12:15, Room A

Let M_n be the algebra of $n \times n$ complex matrices and let $J = I_r \oplus -I_{n-r} \in M_n$, $0 \leq r \leq n$. Consider the indefinite inner product $[\cdot, \cdot]$ defined by $[x, y] = y^* Jx$, $x, y \in \mathbb{C}^n$. A matrix $A \in M_n$ is said to be J-normal if $A^{\#}A = AA^{\#}$, in which $A^{\#}$ is the J-adjoint of A defined by $[Ax, y] = [x, A^{\#}y]$ for any $x, y \in \mathbb{C}^n$ (that is, $A^{\#} = JA^*J$).

A matrix B of size $m \times m$, $m < n$, is said to be imbeddable in $A \in M_n$ if there exists a matrix V of size $n \times m$ such that $V^{\#}V = I_m$ and $V^{\#}AV = B$.

Let $J' = I_{r-p} \oplus -I_{n-r-q}$, with $0 \leq p \leq r$, $0 \leq q \leq n-r$. In this talk we consider the following problem: give necessary and sufficient conditions for a J'-normal matrix $B \in M_{n-p-q}$ to be imbeddable in a J-normal matrix $A \in M_n$. We present an answer to this problem in some particular cases. When $n = r$, the matrix A is normal and the problem was solved by Fan and Pall (1957) for $q = 1$. When $n = r$ and A has real eigenvalues, then A is Hermitian and the answer to the problem is given by the well known interlacing relations for the eigenvalues of A and B.

Joint work with N. Bebiano and J. Providencia (University of Coimbra)

A refined Young inequality and related results
S. Furuichi, Nihon University, Tokyo, Japan
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Mon 17:35, Room C

In this talk, we study on refinements of some inequalities related to Young inequality for scalar and for operator. As our main results, we show refined Young inequalities for two positive operators. Our results refine the ordering relations among the arithmetic mean, the geometric mean and the harmonic mean. Moreover, we give supplements for refined Young inequalities for two positive real numbers. And then we also give operator inequalities based on the supplemental inequalities.

In addition, (if we have an enough time to talk), we show two type of the reverse inequalities of the refined Young inequality for two positive operators, applying the reverse inequalities of the refined Young inequality for positive real numbers

Our talk is based on our recent results [1,2].

[1] S. Furuichi and M. Lin, On refined Young inequalities, arXiv:1001.0195.
[2] S. Furuichi, Reverse inequalities for a refined Young inequality, arXiv:1001.0535.

Joint work with Minghua Lin (University of Regina)

Disturbance decoupling for singular systems by feedback and output injection
M. I. García-Planas, Universitat Politècnica de Catalunya, Spain
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Tue 15:00, Room A

We study the disturbance decoupling problem for linear time invariant singular systems. We give necessary and sufficient conditions for the existence of a solution to the disturbance decoupling problem with or without stability via a proportional and derivative feedback and proportional and derivative output injection that also makes the resulting closed-loop system regular and/or of index at most one. All results are based on canonical reduced forms that can be computed using a complete system of invariants.

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[2] D. Chu and V. Mehrmann, *Disturbance Decoupling for Descriptor Systems by state feedback.* Siam J. Control Optim. vol. 38 (6), pp. 1830-1858, (2000).
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Full Rank Factorization with Quasy Neville Elimination Process
Maria T. Gassó, Instituto de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Spain
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Thu 15:25, Room Galilei

Let A be a real $m \times n$ matrix with $rank(A) = p$. A decomposition $A = LS$ is called full rank factorization of A, if $L \in R^{m \times p}$,

$S \in R^{p\times n}$ and $rank(L) = rank(S) = p$. Several authors have studied different classes of matrices obtaining properties and characterizations of them in terms of full rank factorizations. Recently, Cantó et al. (see [1]) obtain a characterization of *tn* (totally negative) and *tnp* (totally nonpositive) matrices in terms of their full rank factorization in echelon form.

In [2] the authors introduced a variant of the Neville elimination process, *Quasi-Neville elimination*, which consists of leaving the zero row in its position and continuing the elimination process with the matrix obtained from A by deleting the zero rows. The essence of this process is to use the property N introduced by M. Gasca and J.M. Peña in [3]: *An $n \times m$ real matrix A satisfies the condition N if whenever we have carried some rows down to the bottom in the Neville elimination of A, those rows were zero rows, and the same condition is satisfied in the Neville elimination of U^T.* In this work we introduce a new class of matrices weakening the N condition.

Definition. Let A be an $m \times n$ real matrix. A satisfies the condition WN if A satisfies the property N only for rows.

By applying Quasy Neville elimination process we can prove the following result.

Theorem. Let A be an $m\times n$ real matrix, with $rank(A) = p$ and satisfying the WN condition. Then A admits a full rank factorization in echelon form

$$A = LS,$$

where L is a lower echelon matrix of size $m \times p$, S is an upper echelon matrix of size $p \times n$ and $rank(L) = rank(S) = p$.

From this result we obtain a full rank factorization in echelon form of a class of matrices that contains the classes of *tn*, *tnp*, TP (totally positive), TNN (totally nonnegative) matrices. This class also includes the sign regular matrices introduced in [4], some Vandermonde matrices and the semiseparable matrices.

Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 2 & 4 & 5 \\ 1 & 3 & 2 & 4 & 6 \end{bmatrix}.$$

We can prove that this matrix satisfies the WN condition, but does not satisfy the N condition. In addition, we can observe that this matrix is neither TP, TNN, *tn* nor *tnp*.

[1] R. Cantó, B. Ricarte and A.Urbano, Full rank factorization in echelon form of totally nonpositive (negative) rectangular matrices, Linear Algebra and its Applications, DOI: 10.1016/j.laa.2009.07.020.
[2] M. Gassó and Juan R. Torregrosa, A totally positive factorization of rectangular matrices by the Neville elimination, SIAM Journal Matrix Anal. Applications, 25(4) pp. 986-994, 2004.
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Joint work with M. Abad and Juan R. Torregrosa (Instituto de Matemática Multidisciplinar, Universidad Politécnica de Valencia, Spain)

The Padé iterations for the matrix sign function and their reciprocals are optimal

F. Greco, Università di Perugia, Italy
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Tue 15:25, Room Fermi

Rational iterations of the form $z_{k+1} = \varphi(z_k)$, for some rational function $\varphi(z) = a(z)/b(z)$, having attractive fixed points at 1 and -1 locally converge to the sign function and thus they can be used to compute important matrix functions such as the matrix sign function and the matrix square root.

We show that among the rational iterations locally converging with order $s > 1$ to the sign function, the ones belonging to the Padé family and their reciprocals are the unique with the lowest sum of the degrees of numerator and denominator.

This provides a good motivation for their choice in numerical computation.

Joint work with B. Iannazzo (Università di Perugia) and F. Poloni (SNS, Pisa)

Perturbation theory for block operator matrices and applications

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Thu 11:50, Room A

Block operator matrices are matrices whose entries are linear operators on Hilbert or Banach spaces. Both bounded and unbounded operators are allowed as matrix entries. Such objects have found various applications—over the last 20 years—in both applied as well as theoretical mathematics. However, until an excellent recent monograph of C. Tretter their spectral theory has not found its way into standard textbooks. We refer an interested reader to the monograph for a extensive review of the relevant literature. In this talk we present new results in the relative perturbation theory for unbounded block operator matrices. As a first application we discuss adaptive finite element eigenvalue methods from the viewpoint of the spectral theory of block operator matrices. As a result we obtain robust reliability and efficiency estimates for the eigenvalue and eigenvector estimation. Furthermore, we introduce the notion of the enhanced Ritz value and show that it can be used to obtain new computable eigenvalue enclosures which are sharper than those which can be obtained from Ritz values. To illustrate the versatility of the block operator matrix approach we briefly and informally show an application of our results in the spectral theory of operator realizations of elliptic systems of partial differential equations. The basic flavor of the whole theory is a notrivial application of standard results from Linear Algebra in the rigorous theory of elliptic partial differential equations.

A Null Space Free Jacobi-Davidson Iteration for Three Dimensional Photonic Crystals

Tsung-Ming Huang, Department of Mathematics, National Taiwan Normal University, Taipei, 116, Taiwan
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Thu 15:50, Room C

We present an efficient null space free Jacobi-Davidson method to compute the positive eigenvalues of the degenerate elliptic operator arising from Maxwell's equations. We consider spatial compatible discretizations such as Yee's scheme which guarantee the existence of a discrete vector potential. During the Jacobi-Davidson iteration, the correction process is applied to the vector potential instead. The correction equation is solved approximately as in standard Jacobi-Davidson approach. The computational cost of the transformation from the vector potential to the corrector is negligible. As a consequence, the expanding subspace automatically stays out of the null space and no extra projection step is needed. This new method is mathematically equivalent to the standard Jacobi-Davidson method for solving the corresponding generalized eigenvalue problem but the expanding subspace automatically stays out of the null space. Numerical evidence confirms that the new method is much more efficient than the standard Jacobi-Davidson method.

Joint work with Yin-Liang Huang (National Taiwan University), Wen-Wei Lin (National Chiao Tung University) and Wei-Cheng Wang (National Tsing Hua University)

H-expansive matrices in indefinite inner product spaces and their invariant subspaces

DB Janse van Rensburg, North-West University, Potchefstroom, South Africa
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Tue 11:00, Room A

We consider indefinite inner products given by a square real invertible symmetric matrix $H = H^T : [x, y] = (Hx, y)$, [1]. On the Euclidean space equipped with this indefinite inner product, we consider matrices A for which $A^*HA - H$ is nonnegative definite. Such matrices are called H-expansive matrices.

We are interested in the construction of A-invariant maximal H-nonnegative and nonpositive subspaces. The complex case has already been treated by means of a suitable Cayley transform, [2]. The problem when A is real and A has both 1 and -1 as eigenvalues cannot be treated in a straightforward way by means of Cayley transform. We propose a more direct approach. The uniqueness and stability of these subspaces are also studied.

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Joint work with J.H. Fourie (NWU, Potchefstroom, SA), G. Groenewald (NWU, Potchefstroom, SA), A.C.M. Ran (VU, Amsterdam, NL)

The Development of Excel and Sage Math tools for Linear Algebra

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Fri 15:25, Room A

It has been well-known that applications of technology is getting more important for our Linear Algebra class. In particular, MS Excel and Sage Math have been a powerful tools on E-learning environment of today. We will introduce what we have done on the developement of MS Excel tools and Sage Math tools for our Linear Algebra class. We would like share our experiences in this talk.

On a confluent Vandermonde matrix polynomial

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Mon 15:50, Room A

In a paper, [1], the null space of a Vandermonde matrix polynomial of block Toeplitz type has been studied. This was of relevance for the characterization of a matrix polynomial equation having non-unique solutions. The origin of the problem can be retrieved in [2], where an interconnection between the Fisher information matrix of an ARMAX process and a solution to a Stein equation is established. We shall now consider the problem by embedding the results in [1] in a much more general approach to obtain properties of matrix polynomials that can be viewed as generalizations of a confluent Vandermonde matrix.

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Joint work with Peter Spreij (University of Amsterdam)

Solvability of regular pencils for quadratic inverse eigenvalue problem

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In this paper, we are interested in the study of solvability of the quadratic inverse eigenvalue problem (QIEP) of dimension n. Let $k_* = (1 + \sqrt{1 + 8n})/2$ and $0 \leq k < k_*$, and for $m := n + k$ prescribed eigenpairs $\{(\lambda_j, x_j)\}_{j=1}^{m}$, we prove that, generically, there is a constructible nonsingular symmetric quadratic pencil solution $Q(\lambda) \equiv \lambda^2 M + \lambda C + K$ to the QIEP such that $Q(\lambda_j)x_j = \mathbf{0}$ $(j = 1, \ldots, m)$. If $k_* \leq k \leq n$, we show that, generically, all symmetric quadratic pencil solutions are singular. We also derive the dimension of the solution subspace of the QIEP for both cases. Furthermore, we develop an algorithm for finding a symmetric positive definite M for the QIEP if it exists.

Joint work with Yunfeng Cai (Peking University), Wen-Wei Lin (National Chiao Tung University) and Shu-Fang Xu (Peking University)

Jordan orthogonality homomorphisms on Hermitian matrices.

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Mon 12:15, Room B

One of the products to consider on complex Hermitian matrices is the Jordan product $AB + BA$. We say that two Hermitian matrices are Jordan-orthogonal if their Jordan product vanishes. Additive maps which preserve Jordan orthogonality on Hermitian matrices and their infinite-dimensional counterpart, i.e., self-adjoint operators, have recently been investigated by Hou and Zhao [1] on self-adjoint operators and by Chebotar, Ke and Lee [2] on matrix rings with involution.

In applications it is imperative to obtain strong structural results with the minimum possible assumptions. We thereby classify homomorphisms of Jordan orthogonality on Hermitian matrices, i.e. we classify maps (without additivity or bijectivity assumptions) with the property that $AB + BA = 0$ implies the same condition on the images of matrices. We also added one rather small, but unavoidable, technical assumption that no nonzero matrix is annihilated. With such a limited restrictions on map we can not hope for a nice structural result on the whole Hermitian matrices. Nonetheless, with the help of our results we could show that every nonconstant homomorphism of Jordan product on Hermitian matrices (again nor additivity nor bijectivity is assumed) is automatically a linear Jordan isomorphism.

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Joint work with A. Fošner (IMFM Slovenia), N.-S. Sze (Dept. of Applied Math., The Hong Kong Polytechnic University, Humg Hom, Hong Kong), and T. Kuzma

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Trace Inequalities for Logarithms and Powers of J-Hermitian Matrices

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Mon 17:10, Room C

Some spectral inequalities are presented for the trace of logarithms, exponentials and powers of J-Hermitian matrices, $J = I_r \oplus -I_{n-r}, 0 < r < n$. These inequalities are established in the context of indefinite inner product spaces and they are known to be valid for Hilbert space operators or operator algebras.

Key words: Indefinite inner product space, J-Hermitian matrix, relative entropy, Tsallis entropy, Klein inequality, Peierls-Bogoliubov inequality.

Joint work with N. Bebiano (University of Coimbra), J. Providência (University of Coimbra), G. Soares (University of Trás-os-Montes e Alto Douro)

Integer partitions and linear systems over the ring of real continuous functions defined on the unit circle

M. M. López-Cabeceira, University of León, Spain
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Thu 15:00, Room B

A locally Brunovsky linear system is a reachable linear system locally feedback equivalent to a classical canonical form.
Let R be the ring of real continuous functions defined on the unit circle. We describe an algorithm for generating all locally Brunovsky classes over R and give a bound of the number of such classes through integer partitions with special conditions.

Generalized Krein Conditions on the Parameters of a Strongly Regular Graph

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Thu 15:50, Room A

Let X be a strongly regular graph with three distinct eigenvalues. We associate a three dimensional Euclidean Jordan algebra V to the adjacency matrix of X. Then we generalize the Krein parameters of a strongly regular graph and obtain some generalized Krein admissibility conditions for strongly regular graphs.

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Joint work with Domingos Cardoso (University of Aveiro, CEOC), Enide Martins (University of Aveiro, CEOC), Luis Vieira (University of Porto, CMUP)

Accurate eigenvalues of Said-Ball-Vandermonde matrices

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Said-Ball-Vandermonde matrices are a generalization of the Vandermonde matrices arising when the power basis is replaced by the Said-Ball basis. When the nodes are inside the interval $(0, 1)$, then those matrices are strictly totally positive [1]. In this work an algorithm for computing the bidiagonal decomposition of those Said-Ball-Vandermonde matrices is presented, which allows us to use known algorithms for computing the eigenvalues of totally positive matrices represented by their bidiagonal decomposition [2]. The algorithm is shown to be fast and to guarantee high relative accuracy. Some numerical experiments which illustrate the good behaviour of the algorithm are included.

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Joint work with J. J. Martínez (University of Alcalá)

Multi-way adaptive solution of parametric PDE eigenvalue problems

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Mon 15:00, Room Fermi

In this talk we present a multi-way adaptive method for parametric eigenvalue problems. A posteriori error estimates for eigenvalues and associated eigenfunctions for both self- and non-selfadjoint problems will be introduced. These estimates take into account an inexact solution of the corresponding algebraic eigenvalue problem. In our adaptive algorithm we balance the discretization and algebraic error, and introduce the efficient stopping criteria. Additionally, for the non-selfadjoint problem we discuss a new computational procedure based on the adaptive homotopy approach. This is partially a joint work with C. Carstensen, J. Gedicke (HU Berlin, Germany).

Joint work with Volker Mehrmann (TU Berlin, Germany)

On the growth factor for generalised orthogonal matrices

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Mon 11:00, Room A

When Gaussian elimination is applied on a completely pivoted (CP) matrix A the growth factor is defined as $g(n, A) = \frac{\max\{p_1, p_2, \ldots, p_n\}}{|a_{11}|}$, where $p_1, p_2, \ldots, p_n$ are the pivots of A. In 1968 Cryer [1] formulated the following conjecture.

$$g(n, A) \leq n, \text{ with equality iff } A \text{ is a Hadamard matrix.}$$

We will describe the progress on the equality part of this conjecture by presenting all the results concerning the growth factor for Hadamard matrices H_n of dimension n, for binary Hadamard matrices and for weighing matrices of order n and weight k. The latest matrices can achieve moderate growth factor. All these matrices are special cases of generalised orthogonal matrices. Also we will develop theoretical methodologies [3,4] computing the minors of the above type matrices which can lead to numerical algorithms evaluating their pivots. A great difficulty arises at the study of this problem because the pivot pattern is not invariant under H-equivalence. In [2] the unique pivot pattern for H_{12} was presented and in [5] all 34 possible pivot patterns of H_{16} were demonstrated theoretically and the complete pivoting conjecture for H_{16} was proved. The determination of the pivot patterns for H_{20} and for higher dimensions remains open.

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[2] A. Edelman and W. Mascarenhas, On the complete pivoting conjecture for a Hadamard matrix of order 12, Linear Multilinear Algebra,38,pp. 181-187, 1995.
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[4] C. Kravvaritis and M. Mitrouli, Evaluation of Minors associated to weighing matrices, Linear Algebra Appl. 426 pp. 774-809, 2007.
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High Relative Accuracy Implicit Jacobi Algorithm for the SVD

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Mon 15:25, Room Fermi

We prove that a Jacobi-like algorithm applied implicitly on a decomposition $A = XDY^T$ of a matrix A, where D is diagonal, and X, Y are well conditioned, computes all singular values of A to high relative accuracy. The relative error in every eigenvalue is bounded by $O(\epsilon \max[\kappa(X), \kappa(Y)])$, where ϵ is the machine precision and $\kappa(X) = \|X\|_2\|X^{-1}\|_2, \kappa(Y) = \|Y\|_2\|Y^{-1}\|_2$ are, respectively, the spectral condition number of X and Y. The singular vectors are also computed accurately in the appropriate sense. We compare it with previous algorithms for the same problem [1] and see that the new algorithm is faster and more accurate. This work is an extension of the Jacobi implicit algorithm presented in [2] for the symmetric eigenproblem.

[1] J. Demmel et al., Linear Algebra and its Applications, 299 (1999) 21-80
[2] F. M. Dopico, P. Koev and J. M. Molera, Numer. Math. 113 (2009) 519-553

Joint work with Froilán M. Dopico and Johan Ceballos (Universidad Carlos III de Madrid, Spain)

Tensor approach to mixed high-order moments of absorbing Markov chains

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Thu 15:50, Room B

Moments of an absorbing Markov chain are considered. First moments and non-mixed second moments of the number of visits are determined in classical textbooks such as the book of J. Kemeny and J. Snell "Finite Markov Chains". The reason is that the first moments and the non-mixed second moments can be easily expressed in a matrix form using the fundamental matrix of the absorbing Markov chain. Since the representation of the mixed moments of higher orders in a matrix form is not straightforward, if ever possible, they were not calculated. The gap is filled now. Tensor approach to the mixed high-order moments is proposed and compact closed-form expressions for the moments are discovered.

Computing Low Rank Approximations of Tensors

Mechie Nkengla, University of Illinois Chicago, USA
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Mon 12:15, Room A

We work on reliable and efficient algorithms for the best low rank approximation and decompositions of tensors. By exploring non-orthogonality required decompositions such as a CUR-like method for tensor, we investigate the concept of whether unfolding (or matricization) in a particular mode makes a difference in the approximation scheme by experimentation on very large data sets. We also show that by padding the large data set with zeroes, the computational cost of the least-square algorithm is improved. Heuristic based methods such as this provide an advantage in terms of computational complexities and we show that even without an intrinsic bound on the approximation error, the approximations are quite acceptable.

Joint work with Shmuel Friedland (University of Illinois at Chicago)

Exploiting structures in palindromic polynomial eigenvalue problems

Vanni Noferini, University of Pisa, Italy
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Fri 17:35, Room Fermi

Representing palindromic matrix polynomials in the Dickson polynomial basis leads to structured generalized eigenvalue problems. We propose a linearization of the latter problem by means of a suitable matrix pencil having rank-structured block coefficients. We discuss a stragegy, based on the QZ method, to exploit this rank-structure and to tackle the associated nonlinear eigenvalue problem.

Commuting and noncommuting graphs of matrices over semirings

Polona Oblak, University of Ljubljana, Slovenia
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Thu 17:35, Room C

The *commuting graph* $\Gamma(S)$ of a set S is the graph, whose vertex set is the set of all noncentral elements of S and $x - y$ is an edge in $\Gamma(S)$ if $xy = yx$ and $x \neq y$. Its complement is called the *noncommuting graph* of S.

In the talk, we give diameters and girths of the commuting and noncommuting graphs of certain subsets of matrices over semirings, namely for the set of nilpotent matrices, invertible matrices, noninvertible matrices and the full matrix semiring.

Joint work with David Dolžan (University of Ljubljana)

PageRank and Social Competences on Social Network Sites

F. Pedroche, Institut de Matemàtica Multidisciplinària. Universitat Politècnica de València. Espanya.
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Wed 12:15, Room A

In this communication a new method to classify the users of an SNS (Social Network Site) into groups is shown. The method is based on the PageRank algorithm. *Competitivity groups* are sets of nodes that compete among each other to gain PageRank via the *personalization vector*. Specific features of the SNSs (such as number of friends or activity of the users) can modify the ranking inside each *Competitivity group*. We call these features *Social Competences*. Some numerical examples are shown.

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Computation of Canonical Forms and Miniversal Deformations of Bimodal Dynamical Systems

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Tue 17:35, Room B

Canonical forms for controllable bimodal dynamical linear systerms (BDLS) have been used by different authors. The uncontrollable case appears naturally, for example, when considering parametrized families of such systems, where the uncontrollability of some of their members can not be avoided by means of a generic perturbation. Here we provide an algorithm to obtain canonical forms for BDLS, both in the controllable and uncontrollable cases, for $n = 2$ and $n = 3$, which are the most frequent dimensions in the applications. We apply them to compute the miniversal deformation of a triple defining a BDLS, in order to study its local perturbations and bifurcation diagram.

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Joint work with J. Ferrer, M. D. Magret, J. R. Pacha (Universitat Politècnica de Catalunya)

On the spectral radius of non-negative matrices

A. Peperko, University of Ljubljana, Slovenia
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Mon 15:25, Room B

Let $K_1, \dots, K_n$ be (infinite) non-negative matrices that define operators on a Banach sequence space. Given a function $f : [0,\infty) \times \ldots \times [0,\infty) \to [0,\infty)$ of n variables, we define a non-negative matrix $\hat{f}(K_1, \dots, K_n)$ and consider the inequality

$$r(\hat{f}(K_1, \dots, K_n)) \leq \frac{1}{n} \left(r(K_1) + \cdots + r(K_n) \right),$$

where r denotes the spectral radius. We find the largest function f for which this inequality holds for all $K_1, \dots, K_n$. We also obtain an infinite-dimensional extension of the result of Cohen asserting that the spectral radius is a convex function of the diagonal entries of a non-negative matrix.

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Joint work with R. Drnovšek (University of Ljubljana, Slovenia)

Singular two-parameter eigenvalue problems and bivariate polynomial systems

B. Plestenjak, University of Ljubljana, Slovenia
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Mon 17:10, Room Fermi

It is well known that roots of a scalar polynomial $p(x)$ are the eigenvalues of its companion matrix. Therefore, one can apply various numerical methods for the eigenproblem to compute the roots of the polynomial.

We will generalize this approach to bivariate polynomial systems

$$\begin{aligned} p_1(x,y) &= 0, \\ p_2(x,y) &= 0. \end{aligned} \tag{1}$$

It is possible to construct matrices A_i, B_i, and C_i, such that $\det(A_i + \lambda B_i + \mu C_i) = p_i(\lambda, \mu)$ for $i = 1, 2$. The roots of (1) are then the eigenvalues of the *two-parameter eigenvalue problem*

$$\begin{aligned} A_1 x_1 &= \lambda B_1 x_1 + \mu C_1 x_1, \\ A_2 x_2 &= \lambda B_2 x_2 + \mu C_2 x_2. \end{aligned} \tag{2}$$

The dimension of the matrices A_i, B_i and C_i is much larger than the order of the polynomial p_i and the two-parameter eigenvalue problem (2) is singular. Recent results and numerical methods for singular two-parameter eigenvalue problems

[2,3] enable us to compute the finite eigenvalues of (2). Combined with the Jacobi–Davidson approach [1], this might be an alternative when we are interested only in part of the roots that are close to a given target.

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Joint work with A. Muhič (University of Ljubljana)

When several matrices share an invariant cone ?

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Mon 15:00, Room B

We analyze finite families of linear operators $\{A_1, \ldots, A_m\}$ acting in $\mathbb{R}^d$ and sharing a common invariant cone in that space, i.e., there is a closed pointed nondegenerate cone $K \subset \mathbb{R}^d$ such that $A_iK \subset K$ for all $i = 1, \ldots, m$. Special properties of such families have found many applications in the study of joint spectral radii, the Lyapunov exponents, combinatorics, graphs and large networks, etc. Operators with a common invariant cone, act, in some sense, "in the same direction" and inherit most of special properties of matrices with nonnegative entries. We preset a sharp criterion for a finite family of operators to possess a common invariant cone. The criterion reduces the problem to equality of two special numbers that depend on the family. In spite of theoretical simplicity of the criterion, the practical implementation may be difficult because of the high algorithmic complexity of the problem. We show that the problem of existence of a common invariant cone for four matrices with integral entries is algorithmically undecidable. This means that there is no algorithm, which for any family of four matrices with integral entries gives the answer "yes" or "no" within finite time. In particular, this problem is NP-hard. On the other hand, some corollaries of the criterion lead to simple sufficient and necessary conditions for the existence of an invariant cone. Finally, we formulate an approximative analogue of the problem and introduce a "co-directional number" of several matrices. This parameter is close to zero if and only if there is a small perturbation of matrices, after which they get an invariant cone. An algorithm for its computation is presented.

The distance from a matrix polynomial to a prescribed multiple eigenvalue

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Thu 12:15, Room A

The *spectrum* of an $n \times n$ *matrix polynomial* $P(\lambda) = \sum_{j=0}^{m} A_j\lambda^j$ ($\det A_m \neq 0$) is $\sigma(P) = \{\lambda \in \mathbb{C} : \det P(\lambda) = 0\}$. An *eigenvalue* $\lambda_0 \in \sigma(P)$ is called *multiple* if its multiplicity as a zero of $\det P(\lambda)$, that is, its *algebraic multiplicity*, is greater than 1. Moreover, the *geometric multiplicity of* $\lambda_0 \in \sigma(P)$ is the dimension of the null space of matrix $P(\lambda_0)$. In this work, we are interested in perturbations of $P(\lambda)$ of the form $Q(\lambda) = \sum_{j=0}^{m}(A_j + \Delta_j)\lambda^j$, where the matrices $\Delta_j \in \mathbb{C}^{n\times n}$ are arbitrary. In particular, for a scalar $\mu \in \mathbb{C}$, we define a distance from $P(\lambda)$ to μ as a multiple eigenvalue and a distance from $P(\lambda)$ to μ as an eigenvalue with geometric multiplicity κ. Using the singular value decomposition of matrix $P(\mu)$, we compute the first distance and an associated optimal perturbation of $P(\lambda)$. Moreover, for the second distance, we obtain upper and lower bounds, constructing perturbations of $P(\lambda)$ that correspond to the upper bounds. Finally, numerical examples are presented to illustrate and evaluate our results.

Joint work with N. Papathanasiou (National Technical University of Athens)

Minimum polynomials and spaces of matrices with special rank properties

Rachel Quinlan, National University of Ireland, Galway
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Mon 11:50, Room B

Let V be a vector space of dimension n over a field K that admits cyclic extensions of degree n. Then V may be equipped with the structure of a field extension L of K, with cyclic Galois group $\langle\sigma\rangle$ of order n. From Artin's theorem on linear independence of characters it follows that every K-linear endomorphism of V has a unique expression of the form

$$a_0\mathrm{id} + a_1\sigma + \cdots + a_{n-1}\sigma^{n-1}, \quad a_i \in L.$$

Thus the K-linear endomorphisms of L can be identified with "polynomial-type" expressions of degree at most $n-1$ in σ. If $p(\sigma)$ is such an expression, we show that the kernel of the endomorphism corresponding to $p(\sigma)$ is at most equal to the degree of $p(\sigma)$. Furthermore, if $W = \langle a_1, a_2, \ldots a_k\rangle$, we show that up to multiplication by an element of $L^\times$ there is a unique polynomial of degree k in σ that annihilates exactly W. Such a polynomial is given by

$$m_W(\sigma) = \det\begin{pmatrix} a_1 & a_2 & \ldots & a_k & \mathrm{id} \\ \sigma(a_1) & \sigma(a_2) & \ldots & \sigma(a_k) & \sigma \\ \vdots & \vdots & & \vdots & \vdots \\ \sigma^k(a_1) & \sigma^k(a_2) & \ldots & \sigma^k(a_k) & \sigma^k \end{pmatrix}$$

By considering K-endomorphisms of V as L-linear combinations of Galois group elements, it is possible to construct subspaces with special rank properties of the space $M_n(K)$ of $n \times n$ matrices over K, and its subspaces $A_n(k)$ of skew-symmetric matrices and $S_n(k)$ of symmetric matrices. For example if $n = 2m+1$ is odd, it can be shown that $A_n(K)$ contains a chain of subspaces

$$A_n(k) = A_0 \supset A_1 \supset A_2 \supset \cdots \supset A_m = 0.$$

with the property that A_{i+1} has codimension n in A_i and for each $i < m$ the non-zero elements of A_i all have rank exceeding $2i$.

Joint work with R. Gow (University College Dublin)

Integral graphs with regularity constraints

P. Rama, University of Aveiro, Portugal
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Fri 16:45, Room C

Given a graph $G = (V(G), E(G))$, a subset of vertices $\emptyset \neq S \subseteq V(G)$ is a (k,τ)-regular set if S induces a k-regular subgraph in G and every vertex $v \in V(G) \setminus S$ has exactly τ neighbors in S.
We characterize some known classes of integral graphs with

(k, τ)-regular sets corresponding to all distinct eigenvalues and identify some particular integral graphs with this property. We also present some graph operations that generate integral graphs with (k, τ)-regular sets for all distinct eigenvalues from integral graphs with the same property.

Joint work with P. Carvalho (University of Aveiro)

The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J–dilations and canonical forms.

A.C.M. Ran, Department of Mathematics, VU University Amsterdam
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Tue 11:25, Room A

The problem of comparing the operators $T^{[*]}T$ and $TT^{[*]}$ in indefinite inner product spaces has already attracted some attention. One of the motivations was a result stating that a matrix T admits polar decomposition if and only if the canonical forms of $T^{[*]}T$ and $TT^{[*]}$ are the same. In the finite dimensional situation canonical forms of the matrices in question were considered for some special cases in [4]. Later on in [1] a full description was provided. On the other hand, the infinite dimensional case is far from being fully understood. For example, zero can be a singular critical point of one of the operators, while it is in the positive spectrum of the other operator. Further examples can be found in [2], where the notions of regular and singular critical point were studied for the pair $T^{[*]}T$ and $TT^{[*]}$. In this talk we present a method of dilation (reduction) for the operator T, which is quite natural for the study of the properties of $T^{[*]}T$ and $TT^{[*]}$. This construction has its origins in [3], and is similar to a construction implicitly used in [1]. Both the infinite and the finite dimensional case will be discussed, as well as an alternative proof of one of the main results of [1].

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Joint work with M. Wojtylak

On the pole placement problem for singular systems

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Tue 16:45, Room B

Given a singular system with outputs

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned}$$

$E, A \in F^{h\times n}, B \in F^{h\times m}, C \in F^{p\times n}$, and a monic homogeneous polynomial $f \in F[x, y]$, we obtain necessary and sufficient conditions for the existence of a state feedback matrix F and an output injection K such that the state matrix $sE-(A+BF+KC)$ has f as characteristic polynomial, under a regularizability condition on the system.

Joint work with F.C. Silva (University of Lisbon, Portugal)

Partitioned triangular tridiagonalization: rounding error analysis

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Mon 15:50, Room Fermi

We consider a partitioned algorithm for reducing the symmetric matrix A to tridiagonal form, which computes a factorization $PAP^T = LTL^T$ where P is a permutation matrix, L is lower triangular with a unit diagonal and bounded off-diagonal elements, and T is symmetric tridiagonal. We show that such a partitioned factorization is backward stable provided that the corresponding growth factor is not too large (the entries can grow in the factor T). The only slight change with respect to the basic (nonpartitioned) algorithm is in the constant that includes the the size of partition which, on the other hand, allows to exploit modern computer architectures through the use of the level-3 BLAS. Experimental results demonstrate that such algorithm achieves approximately the same level of performance as the blocked Bunch-Kaufman code implemented in Lapack. The Bunch-Kaufman method is also conditionally backward stable (assuming no or moderate growth in triangular factors) making these two main approaches comparable also from the numerical stability point of view.

[1] M. Rozloznik, G. Shklarski and S. Toledo: Partitioned triangular tridiagonalization, to appear in ACM Transactions on Mathematical Software.

Joint work with G. Shklarski and S. Toledo (Tel-Aviv University)

Single-input systems over von Neumann regular rings

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Thu 15:50, Room Galilei

This talk deals with the study of linear systems with scalars in a commutative von Neumann regular ring, i.e. a zero-dimensional ring with no nonzero nilpotents (for example $\mathbb{Z}/(n)$, where n is a squarefree integer). It is shown that a commutative ring is von Neumann regular if and only if any single-input system is feedback equivalent to a special normal form. This normal form, which can be obtained by an explicit algorithm, is associated to a collection of principal ideals which determine completeley the structure of the reachability submodule of the system.

K-hyperbolas and polynomial numerical hulls of normal matrices

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Let $A \in M_n$ be a normal matrix and let $k \in \mathbb{N}$. In this note we introduce the notions "k-hyperbola" and "k-hyperbolic region". The polynomial numerical hull of order k, denoted by $V^k(A)$ is characterized by the intersection of k-hyperbolic regions. Also, the locus of $V^{n-1}(A)$ in the complex plane is determined.

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Joint work with Hamid Reza Afshin, Mohammad Ali Mehrjoofard (Vali-E-Asr University of Rafsanjan, Rafsanjan, Iran)

Spectra and cycles of length m in regular tournaments of order n

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Fri 17:10, Room C

A tournament T is an orientation of a complete graph. A tournament is regular if the out-set and in-set of each vertex have the same number of vertices. Let $c_m(T)$ be the number of cycles of length $m \geq 3$ in T. It is well known that any two regular tournaments of (odd) order n have the same number of cycles of length 3 (M.G. Kendall & B. Babington Smith, 1940). The case of $m = 4$ is more complicated. Let RLT_n be the unique regular locally transitive tournament of order n and DR_n be a doubly-regular tournament of this order. According to the results of U. Colombo (1964) and B. Alspach & C. Tabib (1982), for any regular tournament T of order n, we have $c_4(DR_n) \leq c_4(T) \leq c_4(RLT_n)$. In the present talk, based on matrix methods, we show that $2c_4(T) + c_5(T) = n(n-1)(n+1)(n-3)(n+3)/160$. This implies $c_5(RLT_n) \leq c_5(T) \leq c_5(DR_n)$ for any regular tournament T of order n.

Note that if $m = 3, 4, 5$, then $mc_m(T)$ is equal to the trace $tr_m(T)$ of the mth power of the adjacency matrix of T. While $6c_6(T)$ does not equal $tr_6(T)$, we show that $c_6(T)$ is also a function of the spectrum of a regular tournament T. The corresponding pure spectral expression for $c_6(T)$ allows us to prove the inequality $c_6(T) \leq c_6(DR_n)$ with equality holding if and only if $T = DR_n$ or $T = Kz_7$. We also determine the value of $c_6(RLT_n)$ and conjecture that this value is the minimum number of 6-cycles in the class of regular tournaments of order n. Finally, we determine the coefficients at n^m and n^{m-1} in the expansion of the expression for both $c_m(DR_n)$ and $c_m(RLT_n)$ in the case of arbitrary length $m > 3$. This allows us to state that for a sufficiently large order n, $c_m(RLT_n) < c_m(DR_n)$ if $m \equiv 1, 2, 3 \mod 4$, and $c_m(RLT_n) > c_m(DR_n)$ if $m \equiv 0 \mod 4$. In particular, the last inequality means that DR_n cannot be a maximizer of $c_m(T)$ for each $m \geq 5$.

Adjacency preserving maps

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Tue 11:25, Room B

Two matrices are said to be adjacent if one is a rank one perturbation of the other. The classical Hua's theorems characterize bijective maps on various matrix spaces preserving adjacency in both directions. We will present some recently obtained improvements of these results.

Spectral analysis of inexact constraint preconditioning for saddle point matrices

D. Sesana, University of Insubria, Como - Italy
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Fri 11:50, Room Pacinotti

Linear systems with nonsingular coefficient matrix of the form $\mathcal{A} = [A, B^T; B, 0]$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, arise in many applications associated with the numerical solution of saddle point problems. We consider the case where $\mathcal{A}$ is symmetric and highly indefinite, so that preconditioning is mandatory if the associated system is to be solved with a Krylov subspace method. In *constrained* optimization problems, preconditioners of the form

$$\mathcal{P} = \begin{bmatrix} I_n & 0 \\ BD^{-1} & I_m \end{bmatrix} \begin{bmatrix} D & 0 \\ 0 & -H \end{bmatrix} \begin{bmatrix} I_n & D^{-1}B^T \\ 0 & I_m \end{bmatrix},$$

where D and H approximate A and $BD^{-1}B^T$, respectively, are particularly effective. In several applications it is sufficient to choose D to be a scaled multiple of the identity, so that efforts focus on the approximation to $BD^{-1}B^T$, which often cannot be computed explicitly [5].

In this talk we present a spectral analysis of the preconditioned matrix $\mathcal{P}^{-1}\mathcal{A}$ as H moves away from its ideal and computationally expensive version $H_{\text{ex}} = BD^{-1}B^T$. Much is known about the spectrum in the ideal case, characterized by a rich spectral structure, with non-trivial Jordan blocks and favourable real eigenvalue distribution [3, 4]. The spectral analysis of the general though far more realistic case $H \neq BD^{-1}B^T$ has received less attention (see, e.g., [1, 2, 5]), possibly due to the difficulty of dealing with Jordan block perturbations. We show that a two-step procedure allows one to successfully handle this complex structure, revealing the true spectral perturbation induced by a workable choice of H.

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Joint work with V. Simoncini (Università di Bologna)

Jacobi-type algorithms for the Hamiltonian eigenvalue problem

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Mon 17:35, Room Fermi

We present recent results on Jacobi-type algorithms for real Hamiltonian matrices. We describe both, real and complex algorithms. The algorithms use orthogonal (unitary) and non-orthogonal shear transformations. Convergence and accuracy properties of the algorithms are discussed.

Computational aspects of the Moore-Penrose inverse

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Fri 15:00, Room Galilei

In the last years a number of fast algorithms for computing the Moore-Penrose inverse of structured and block matrices have been designed. There is a variety of new papers dealing with numerical algorithms, whose authors neglect the issue of numerical stability of their algorithms and focus only on complexity (number of arithmetic operations). However, very often they are not accurate up to the limitations of data and conditioning of the problem.

In this talk we present a comparison of certain direct and iterative algorithms for computing the Moore-Penrose inverse, from the point of view of numerical stability and algebraic complexity.

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Joint work with Iwona Wróbel (Warsaw University of Technology).

Solving large-scale nonnegative least-squares

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Fri 15:50, Room Galilei

We study the fundamental problem of nonnegative least squares. This problem was apparently introduced by Lawson and Hanson [1] under the name NNLS. As is evident from its name, NNLS seeks least-squares solutions that are also nonnegative. Owing to its wide-applicability numerous algorithms have been derived for NNLS, beginning from the active-set approach of Lawson and Hanson [1] leading up to the sophisticated interior-point method of Bellavia et al. [2]. We present a new algorithm for NNLS that combines projected subgradients with the non-monotonic gradient descent idea of Barzilai and Borwein [3]. Our resulting algorithm is called BBSG, and we guarantee its convergence by exploiting properties of NNLS in conjunction with projected subgradients. BBSG is surprisingly simple and scales well to large problems. We substantiate our claims by empirically evaluating BBSG and comparing it with established convex solvers and specialized NNLS algorithms. The numerical results suggest that BBSG is a practical method for solving large-scale NNLS problems.

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Joint work with D. Kim and I. S. Dhillon (University of Texas at Austin)

The Sinkhorn-Knopp Fixed Point Problem with Patterned Matrices

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Thu 17:10, Room B

We consider the Sinkhorn-Knopp Fixed Point Problem

$$(A^T(A\vec{x})^{(-1)})^{(-1)} = \vec{x}$$

where (-1) is the entry-wise inverse of a vector. This problem arises from work originally done in the 1960s by Sinkhorn and Knopp [1] for transforming a matrix into a doubly stochastic matrix by the pre- and post-multiplication of a matrix by diagonal matrices: D_1AD_2. This process has a variety of applications, including most recently in web page rankings, e.g. in a Google search. We have investigated the types of solutions that arise in solving this fixed point equation both in the general case and for specific types of matrices, in particular, patterned matrices. The results in the circulant case are particularly interesting and exhibit a very cyclical behavior. We share results for the circulant case and other cases involving various other types of patterned matrices, and relate our results to the original problem of trying to transform a matrix into a doubly stochastic one. The majority of this work was done by undergraduates under my supervision.

[1] Richard Sinkhorn and Paul Knopp, *Concerning nonnegative matrices and doubly stochastic matrices*, Pacific J. Mathematics **21** (1967), pp. 343 - 348.

Spectraloid operator polynomials, the approximate numerical range and an Eneström-Kakeya theorem in Hilbert space

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Fri 11:00, Room Galilei

We study a class of operator polynomials in Hilbert space, which are spectraloid in the sense that spectral radius and numerical radius coincide. The focus is on the spectrum in the boundary of the numerical range. As an application the Eneström–Kakeya–Hurwitz theorem on zeros of real polynomials is generalized to Hilbert space.

Joint work with H. K. Wimmer (Universität Würzburg, Germany)

Bifurcation analysis of eigenvalues of polynomial matrices smoothly depending on parameters

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Thu 11:25, Room A

Let $P(\lambda) = \sum_{i=0}^{k} \lambda^i A_i(p)$ be a family of monic polynomial matrices smoothly dependent on a vector of real parameters $p = (p_1, \ldots, p_n)$. In this work we study behavior of an eigenvalue of the monic polynomial family $P(\lambda)$.

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Joint work with M. I. García-Planas (Universitat Politècnica de Catalunya)

A class of matrices generalizing the idempotent ones

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Tue 11:50, Room B

In the last years, the concept of idempotency, tripotency and, in general, $\{k+1\}$-potency has been studied from a different point of view in the literature. For example, the case when linear combination of two idempotent (tripotent, $\{k+1\}$-idempotent) matrices is idempotent (tripotent, $\{k+1\}$-idempotent) has been developed [1,2,3]. It seems to be natural to extend the idea of $\{k+1\}$-potency. In this work, we present such a generalization and we study this kind of matrices giving some properties of them.

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Joint work with L. Lebtahi (Universidad Politécnica de Valencia)

On efficient numerical approximation of the scattering amplitude

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Tue 15:50, Room B

This talk presents results on efficient and numerically well-behaved estimation of the scalar value c^*x, where c^* denotes the conjugate transpose of c and x solves the linear system $Ax = b$, $A \in \mathbb{C}^{N\times N}$ is a nonsingular complex matrix and b and c are complex vectors of length N. In other words, we wish to estimate the *scattering amplitude* $c^*A^{-1}b$.

In our understanding, various approaches for numerical approximation of the scattering amplitude can be viewed as applications of the general mathematical concept of *matching moments model reduction*, formulated and used in applied mathematics by Vorobyev in his remarkable book [3]. Using the Vorobyev moment problem, matching moments properties of Krylov subspace methods can be described in a very natural and straightforward way, see [1]. This talk further develops the ideas from [1] into efficient estimates of $c^*A^{-1}b$, see [2].

We briefly outline the matching moment property of the Lanczos and Arnoldi algorithms, and specify techniques for estimating $c^*A^{-1}b$ with A non-Hermitian, including a new algorithm based on the BiCG method. We show its mathematical equivalence to the existing estimates which use a complex generalization of Gauss quadrature, and discuss its numerical properties. The proposed estimate will be compared with existing approaches using analytic arguments and numerical experiments on a practically important problem that arises from the computation of diffraction of light on media with periodic structure.

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Joint work with Z. Strakoš (Czech Academy of Sciences, Czech Republic)

Computation of the greatest common divisor of polynomials through Sylvester matrices and applications in image deblurring

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Fri 15:50, Room B

We present new, fast methods computing the greatest common divisor (GCD) of polynomials. We develop algorithms computing in an efficient way the upper triangularization of the Modified Sylvester (MS) [4] and Modified Generalized Sylvester (MGS) matrix, resulting to the GCD of polynomials. All methods are exploiting the special structure of MS and MGS matrices, reducing by one order the required complexity in case of several polynomials. For this case, we propose also a fast parallel method for the computation of their GCD using Housholders' transformations, improving significant the complexity of the classical procedure. The use of floating point arithmetic can lead to incorrect GCDs. We study the behavior of the numerical implementation of the methods in respect of the inner accuracy ϵ_t of the procedures and the significant digits that are used. Various values of these quantities many times can lead to different GCDs or comprimeness [2]. The complexity is computed for all methods and tables comparing them with the known techniques [1,3] are given. All the algorithms are tested for several sets of polynomials and the results are summarized in tables, resulting to useful conclusions. Finally, an interesting application in image deblurring is given.

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Joint work with M. Mitrouli (University of Athens)

A preconditioning approach to the Google pagerank computing problem

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Wed 12:40, Room A

It is well known that the Google pagerank vector p can be computed by solving a sparse linear system $Ax = b$. In this talk we first show that such system can be solved by the Euler-Richardson (ER) method with the same computational complexity of the power method (which is the standard algorithm for computing p). Then we observe that, by the particular structure of the Google matrix, only one eigenvalue of A is responsible of the low rate of convergence of ER, and that such eigenvalue can be removed by preconditioning A. In fact, applying ER to $\mathcal{A}^{-1}Ax = \mathcal{A}^{-1}b$ for a suitable choice of the preconditioner $\mathcal{A}$ improves the convergence rate roughly from 0.85^k to 0.25^k. Further studies to reduce the surplus of operations per step are in progress.

Joint work with Carmine Di Fiore (difiore@mat.uniroma2.it)

Extreme Distance Field of Values Points: How to Compute?

Frank Uhlig, Auburn University, Auburn, AL, USA
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Tue 15:25, Room B

The field of values $F(A) = \{x{*}Ax \in \mathbb{C} | x \in \mathbb{C}^n\}$ of a square $n\times n$ matrix A contains highly useful information of A. How can the extreme distance points of $F(A)$ from zero be computed for a given A? Why are these distances important?

In particular, why: the maximal distance, called the numerical radius $r(A)$, of points p in $F(A)$ from zero determines the transient behavior of the system governed by A while a positive minimal distance between zero and $F(A)$, called the Crawford number, insures stability of the system. These related problems have quite different flavors: the spectral radius can be achieved at multiple and even infinitely many points on the boundary of $F(A)$ regardless of where zero lies, but if zero lies outside the field of values then the Crawford number is realized at only one point on the boundary of $F(A)$. And if zero lies inside $F(A)$, then the (negative) generalized Crawford number can occur at any number of $F(A)$ boundary points.

We explain and compare new fast and accurate vector and geometry based algorithms with recent, but much slower optimization type algorithms for these two problems.

Lyapunov Equation Methods for Solving the Matrix Nonlinear Schrödinger Equation

Cornelis van der Mee, Dip. Matematica e Informatica, Università di Cagliari
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Thu 15:25, Room C

We derive exact $n \times m$ matrix solutions of the focusing matrix nonlinear Schrödinger (NLS) equation

$$iu_t + u_{xx} + 2uu^\dagger u = 0, \qquad (x,t) \in \mathbb{R}^2, \tag{1}$$

by considering $u(x,t)$ as the potential in the matrix Zakharov-Shabat system

$$\frac{\partial \Psi}{\partial x}(x,\lambda) = \begin{pmatrix} i\lambda I_n & -u(x,t) \\ u(x,t)^\dagger & -i\lambda I_m \end{pmatrix} \Psi(x,\lambda), \qquad x \in \mathbb{R}, \tag{2}$$

and applying inverse scattering of Eq. (2) by solving the coupled Marchenko integral equations. Exploiting the Hankel structure of its integral kernels and representing them in the form

$$\Omega(x+y) = Ce^{-(x+y)A}e^{4itA^2}B \tag{3}$$

for suitable matrix triplets (A, B, C), exact solutions $u(x,t)$ of Eq. (1) are obtained, requiring a careful analysis of two Lyapunov matrix equations. Next, we discuss various transformations to generate matrix NLS solutions from other such solutions.

Joint work with Francesco Demontis (Università di Cagliari)

Matrix algebras can be spectrally equivalent with ill conditioned Toeplitz matrices

P. Vassalos, Athens University of Economics and Business, Greece
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Fri 11:25, Room Pacinotti

In this work, we prove the existence of matrices, $\tau_n(f)$, belonging to τ algebra that are spectrally equivalent with ill conditioned Toeplitz matrices $T_n(f)$. For that, we assume that the generating function f is real valued, nonnegative, continuous, with isolated roots of maximum order $\alpha \in \mathbb{R}^+$. Specifically, we prove that for $0 \le \alpha \le 2$ there exist a proper clustering of the eigenvalues of $\tau_n(f)^{-1}T_n(f)$ around unity. For $2 < \alpha < 4$, a weak clustering for the spectrum of the aforemention matrix is achieved, where the minimum eigenvalue is bounded from bellow, while a constant number, independent of n, of eigenvalues tend to infinity. The results are generalized to cover the more interesting, from theoretical and practical point of view, case of Block Toeplitz with Toeplitz Blocks (BTTB), matrices. Based on these theoretical statements we propose τ preconditioners that lead to superlinear convergence both in $1D$ and $2D$ case when the condition number of the Toeplitz matrix is $o(n^4)$. Finally, we show that the spectrally equivalence also holds between circulant matrices and ill-conditioned Toeplitz matrices. The main difference is that the continuous symbol which generates the Toeplitz matrix should have discrete roots of order less than 2. We perform many numerical experiments, whose results confirm the validity of theoretical analysis.

Joint work with D. Noutsos (University of Ioannina)

Francis's Algorithm

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Mon 16:45, Room Fermi

Francis's implicitly-shifted QR algorithm has for many years been the most widely used algorithm for computing eigenvalues of matrices. The standard (and time-honored) method of justifying Francis's algorithm is to show that each iteration is equivalent to a step (or several steps) of the explicitly-shifted QR algorithm. In this talk we will argue that the standard approach is unduly complicated. Instead one should argue directly that Francis's algorithm performs nested subspace iterations with a change of coordinate system at each step. This is done by bringing to light the role of the nested Krylov subspaces that lurk in the transforming matrices.

Block diagonalization for a matrix A when $AR = RA$ and $R^k = I$

James R. Weaver, Dept. of Mathematics/Statistics, University of West Florida, Pensacola, USA
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Thu 15:00, Room Galilei

This article examines the block diagonalization of a $n \times n$ complex matrix A when $AR = RA$ for a general $n \times n$ complex matrix $R$with the property that $R^k = I$ for a positive integer k. First find the Jordan Form for the matrix R, which is a diagonal matrix D in the case $R^k = I$, and a corresponding transforming matrix P. This information is used to find a block diagonalization of A given some additional information about the matrix A.

Commutators with maximal Frobenius norm

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Fri 16:45, Auditorium

When investigating the commutator of two normed $n\times n$ matrices, two situations are of special interest: the commuting case (i.e. the commutator is zero) and the "maximal non-commuting case" (in which the commutator has a norm as large as possible). Regarding matrices at random typically yields pairs very close to commutativity – especially if their size n is large. Although actually none of these matrices really commute, except for 2×2 matrices it is seemingly hopeless to find a pair whose commutator admits Frobenius norm $\sqrt{2}$, which is the known bound for the maximal situation.

We will present an explanation for that behaviour by determining all pairs of real or complex matrices satisfying the equality

$$\|XY - YX\|_{\mathsf{F}} = \sqrt{2}\|X\|_{\mathsf{F}}\|Y\|_{\mathsf{F}}.$$

The result is a surprisingly simple and meager, but also nicely structured set. The talk is based on joint work with Che-Man Cheng and Seak-Weng Vong.

[1] C.-M. Cheng, S.-W. Vong, D. Wenzel, Commutators with maximal Frobenius norm, Linear Algebra Appl. 432 (2010) 292–306.

Joint work with Che-Man Cheng and Seak-Weng Vong (University of Macau)

Hyperinvariant, characteristic and marked subspaces

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Mon 11:25, Room B

Abstract: Let V be a finite dimensional vector space over a field K and f a K-endomorphism of V. We study three types of f-invariant subspaces, namely hyperinvariant subspaces, which are invariant under all endomorphisms of V that commute with f, characteristic subspaces, which remain fixed under all automorphisms of V that commute with f, and marked subspaces, which have a Jordan basis (with respect to $f_{|X}$) that can be extended to a Jordan basis of V. We show that a subspace is hyperinvariant if and only if it is characteristic and marked. If K has more than two elements then each characteristic subspace is hyperinvariant.

Joint work with Pudji Astuti (ITB Bandung)

Structured matrix methods for the computation of multiple roots of inexact polynomials

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Fri 15:00, Room B

This paper considers the application of structured matrix methods for the calculation of a structured low rank approximation of the Sylvester resultant matrix of two polynomials that are corrupted by additive noise. It is shown that this low rank approximation allows the computation of multiple roots of inexact polynomials that have been corrupted by additive noise, such that the multiplicities of the theoretically exact roots are preserved. Particular problems occur with polynomials whose coefficients vary widely in magnitude, and it is shown that these polynomials must be processed prior to the computation of their roots.

The talk will contain computational results that demonstrate the theoretical analysis.

Joint work with Madina Hasan (University of Sheffield) and Xin Lao (University of Sheffield)

On the numerical range of companion matrices

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Fri 11:25, Room Galilei

It is known that the convex hull of the roots of a given polynomial contains the roots of its derivative. This result is known as the Gauss-Lucas theorem. We will investigate the possibility of generalizing it to the numerical range of companion matrices and discuss the relation between the numerical ranges of companion matrices of a polynomial and its derivative. Several types of companion matrices will be considered.

Matrix inequalities associated with the data processing inequality

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Mon 16:45, Room C

In the cascade of two channels $X \to Y \to Z$, a refined version of the data processing inequality of the form $\frac{I(X,Z)}{I(X,Y)} \leq c(A)$ has been found by Evans and Schulman [1] for binary channels, where the bound $c(A)$ (≤ 1 generally) depends only on the channel matrix A of the second channel $Y \to Z$. In this report we give a general observation that may help us to find such bounds for non-binary channels, and find one for a certain symmetric A.

[1] W. S. Evans and L. J. Schulman, *Signal propagation and noisy circuits*, IEEE Trans. Inform. Theory **45** (1999), no. 7, 2367–2373.

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