Linear Algebra and Inverse Problems

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Inverse problems arise in many important applications, including medical imaging, microscopy, geophysics, and astrophysics. Because they often involve large scale, extremely ill-conditioned linear systems, linear algebra problems associated with inverse problems are extremely challenging to solve, both mathematically and computationally. Solution schemes require enforcing regularization, using for example prior information and/or by imposing constraints on the solution. In addition, matrix approximations and fast algorithms for structured matrices must be employed. The speakers in this minisymposium will report on recent research developments involving linear algebra aspects of inverse problems, including algorithms and other computational issues.

Nonsmooth/Smoothing Optimization Approaches to Structured Inverse Quadratic Eigenvalue Problems

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Thu 16:45, Room Galilei

Structured inverse quadratic eigenvalue problems arise in the fields of structural dynamics, acoustics, electrical circuit simulation, fluid mechanics, etc. In this talk, we present some nonsmooth/smoothing optimization methods for solving structured inverse quadratic eigenvalue problems. The proposed algorithms are based on the recent developments in strong semismooth matrix-valued functions [1] and strong semismooth eigenvalues of symmetric matrices [2]. The global and locally fast convergence is established. Numerical experiments show the efficiency of the proposed methods.

[1] D. Sun and J. Sun, Semismooth matrix valued functions, Math. Oper. Res., 27, pp. 150-169, 2002.
[2] D. Sun and J. Sun, Strong semismoothness of eigenvalues of symmetric matrices and its application to inverse eigenvalue problems, SIAM J. Numer. Anal. 40, pp. 2352-2367, 2002.

Bayesian Hypermodels for Inverse Problems

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Wed 12:15, Room Galilei

In this talk, I will discuss inverse problems in the context of Bayesian statistics, where the regularization function corresponds to the negative-log of the prior probability density. From the Bayesian perspective, the regularization parameter can be viewed as a hyper-parameter, i.e. as a random variable with some known distribution. Adding this element of uncertainty to the value of the regularization parameter is not only honest, it allows for increased flexibility. For example, one can sample from the posterior regularization parameter distribution, obtaining an empirical density (histogram) and hence confidence intervals for the regularization parameter. One can also allow for the regularization parameter to be spatially dependent (i.e. vector valued), which leads to adaptive methods and Bayesian learning. Numerical examples will be used to illustrate the various concepts.

A survey of scaled gradient projected methods for nonnegative image reconstruction

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Wed 12:40, Room Galilei

In a regularization or Bayesian approach, the ill-posed problem of image reconstruction with nonnegativity constraint is reduced to the constrained minimization of a convex function. Implicit methods have a fast asymptotic convergence rate but require to solve a linear equation per iteration while explicit methods, with a slower convergence, require only matrix-vector multiplications. However, a recently proposed class of scaled gradient projection (SGP) method [1] can provide efficient algorithms, thus proposing these explicit methods as an interesting alternative to the implicit ones. Moreover, in the case of non-regularized minimization these methods still exhibit the *semi-convergence property.*

In this talk, after a general outline of the proposed SGP, we discuss a few applications. The first is to the problem of the nonnegative least-squares solution [3], the second to the denoising of images corrupted by Poisson noise [2] and the third to the non-regularized deblurring of Poisson data [1]. In such a case the SGP algorithm provides an acceleration of the standard EM method. Future applications to the regularized deblurring of Poisson data are also briefly discussed.

[1] S. Bonettini, R. Zanella and L. Zanni, A scaled gradient projection method for constrained image deblurring, Inverse Problems, 25, 015002 (23pp), 2009.
[2] R. Zanella, P. Boccacci, L. Zanni and M. Bertero, Efficient gradient projection methods for edge-preserving removal of Poisson noise, Inverse Problems, 25, 045010 (24pp), 2009.
[3] F. Benvenuto, R. Zanella, L. Zanni and M. Bertero, Nonnegative least-squares image deblurring: improved gradient projection approaches, Inverse Problems, 26, 025004 (18pp), 2010.

Joint work with S. Bonettini (University of Ferrara), R. Zanella and L. Zanni (University of Modena-Reggio Emilia), F. Benvenuto and P. Boccacci (University of Genova)

Designing Optimal Filters for Ill-Posed Inverse Problems

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Thu 17:10, Room Galilei

Filtering methods are essential for computing reasonable solutions to ill-posed inverse problems. Without proper filtering, it is well known that small amounts of noise in the data may amplify, resulting in catastrophic errors in the solution. However, standard filtering methods such as Truncated-SVD and Tikhonov filtering may perform poorly for a given problem or application. In this paper, we are interested in designing optimal filters for a given operator of a given application. Utilizing techniques from stochastical and numerical optimization, we present a novel and efficient approach for constructing optimal filters based on minimizing the expected value of the mean square error estimates. Image deblurring is one application that relies heavily on robust filtering techniques, and numerical examples on testing data illustrate that our proposed filters perform consistently better than well established filtering methods.

Joint work with M. Chung (Emory University) D. P. O'Leary (University of Maryland, College Park)

Structured shift-variant imaging systems and invariant approximations via coordinate transformations

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Wed 11:50, Room Galilei

In the simplest Fredholm equations of the first kind arising in real applications, the integral kernel is shift-invariant, that is, the impulse response does not change as the object position is shifted. In image deblurring, this happens when exactly the same blur covers all the image domain. On the other hand, in the general case the shape of the impulse response might change as the object position is changed, that is, different regions of the image might be subjected to different blurs. These kinds of blurring models, termed as shift-variant, are much more involving since they require high numerical complexity in time and memory. However, many shift-variant integral kernels are intrinsically shift-invariant. We can call them as structured shift-variant. The well known main example is the rotational blur, which arises when the object rotates with respect to the imaging apparatus. Basically, although the blur changes with respect the object position (in particular, it is small close to and increases far from the center of the rotation), if the coordinate system is changed from Cartesian to Polar, then the integral kernel becomes explicitly shift-invariant. In this talk we analyze in a general and algebraic setting these kinds of structured shift-variant imaging systems. In this respect, we propose an algorithm for finding the coordinate transformation which allows a structured shift-variant PSF to become explicitly shift-invariant. The usage of the computed coordinate transformation will highly reduce the numerical complexity of the imaging system. Some numerical results related to a real application in External Vehicle Speed Control will end the talk.

Level set methods for the reconstruction of electrical conductivity by eddy current imaging

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Thu 11:50, Room Galilei

We present a numerical method for solving an inverse problem for the 3D imaging of small, conductive inclusions in an insulating medium from exterior measurements. The method exploits a non-destructive technique based on eddy currents to analyze the response to the field of a probe coil placed at various positions and excited at different frequencies [1].

The computational problem consists of a large, distributed parameter estimation problem, having some peculiarities:

- The numerical evaluation of the forward map of the problem is a very expensive task;
- the sought solution is a piecewise constant function with a quite small support, and whose nonzero values are possibly known a priori;
- the continuous relaxation of the problem is a large, very under-specified, nonlinear problem.

Regularization is introduced as a sort of sparsity constraint on the (discretized) gradient of level set functions [2]. The talk illustrates various linear algebraic issues occurring in the numerical solution of this problem. The efficacy of the obtained method is substantiated by numerical simulations.

[1] A. Pirani, M. Ricci, R. Specogna, A. Tamburrino, F. Trevisan. Multi-frequency identification of defects in conducting media. *Inverse Problems* 24 (2008), 035011, 18 pp.
[2] A. DeCezaro, A. Leitão, X.-C. Tai. On multiple level-set regularization methods for inverse problems. *Inverse Problems* 25 (2009), 035004, 22 pp.

Joint work with R. Specogna (ruben.specogna@uniud.it, University of Udine, Italy) and F. Trevisan (trevisan@uniud.it, University of Udine, Italy)

Edge–Preserving Regularization in Color Image Reconstruction

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Wed 11:25, Room Galilei

Various color image inverse problems can be solved using a regularization technique. In this way the solution is defined as the argument of the minimum of an energy function, given by the sum of two terms. The first term is a consistency data term while the later one is related to the smoothness constraints. The design of the energy function is crucial for a proper image reconstruction. In this paper we focus on the construction of the smoothness term for an edge–preserving color image reconstruction.

Ideal images present intensity color discontinuities in correspondence of sharp color variations. By means of a duality theorem [1], we propose the use of a stabilizer that implicitly deals with line variables. These variable are related to the discontinuities in the intensity field. A correct estimation of the values of the line variables allows a more efficient image reconstruction. Many authors have noted as the high frequencies of the three RBG channels of a ideal image were very similar [2][3], so we propose to add to the smoothness term a new term related to the difference of the finite derivatives in different channels.

For the minimization of the energy function we propose a Graduated Non-Convex (GNC) technique and the experimental results confirm efficiency of the method.

[1] F. Martinelli, Regularization Techniques in Image and Signal Processing, PhD Thesis, University of Perugia, 2009.
[2] B.K. Gunturk, Y. Altunbasak, R.M. Mersereau, Color Plane Interpolation using Alternating Projections, IEEE Transactions on Image Processing, n. 9 vol. 11, pp. 997–1013, 2002.
[3] J. Mairal, M. Elad, G. Sapiro, Sparse Representation for Color Image Restoration, IEEE Transactions on Image Processing, n. 1 vol. 17, pp 53–69, 2008.

Sparse Approximate Inverse Preconditioning for Smoothing and Regularization

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Thu 17:35, Room Galilei

We consider sparse approximate inverses for preconditioning iterative methods. Especially we are interested in applications where the iterative solver should reduce the error only in certain subspaces like in Multigrid or in ill-posed inverse problems. We derive two different methods to compute sparse approximate inverses with different behaviour on high frequency components and low frequency components. The new preconditioners lead to an improved smoothing property in Multigrid and to better reconstruction of the blurred data in ill-posed inverse problems.

Joint work with M. Sedlacek (Technical University Munich)

Edge Preserving Projection-based Regularization

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Thu 11:25, Room Galilei

We present a projection-based regularization strategy and algorithm for retaining edges in a regularized solution. Our algorithm is suitable for large-scale discrete ill-posed problems arising from the discretization of Fredholm integral equations of the first kind; for example, image deblurring in two and three dimensions, the focus of our talk.

Our strategy avoids some of the pitfalls of many other well-known edge-preserving methods by making use of orthogonal decompositions/transforms in which components in the so-called noise and signal subspaces can be generated quickly. In determining the appropriate orthogonal transform, we exploit matrix structure as well as properties of the underlying continuous model. Numerical results show the promise of our approach.

Joint work with Per Christian Hansen (Technical University of Denmark), Donghui Chen (Tufts University)

Iterative methods for Tikhonov Regularization

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Thu 11:00, Room Galilei

The solution of linear discrete ill-posed problems is very sensitive to perturbations in the data. Tikhonov regularization is a popular approach to modifying these problems in order to make them less sensitive. We discuss iterative methods for the solution of large-scale Tikhonov-regularized problems with a general linear regularization operator.

Joint work with Hochstenbach, A Neuman, H. Sadok, F. Sgallari, and Q. Ye.

Multisplitting for Regularized Least Squares

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Thu 12:15, Room Galilei

Least squares problems are one of the most often used numerical formulations in engineering. Many such problems lead to ill-posed systems of equations for which a solution may be found by introducing regularization. The use of multisplitting least squares, as originally introduced by Renaut for well-posed least squares problems, is extended to Tikhonov regularized large scale least squares problems. Regularization at both the global and subproblem level is considered, hence providing a means for multiple parameter regularization of large scale problems. Basic convergence results follow immediately from the original formulation. The iterative scheme to obtain the global solution uses repeated solves of local regularized systems each with a fixed system matrix but updated right hand side. Updates of the underlying Krylov subspace for the multiple right hand side system improve the efficiency of the local solver at each step. Numerical validation is presented for some simple one dimensional signal restoration simulations from the Regularization Toolbox of Hansen. The reconstruction of Shepp-Logan phantom data provides an example for a large scale problem. The use of local regularization parameters is also illustrated for a 1D restoration problem with variable noise in the signal. The implementation of the algorithm with GPUs for image restoration will also be discussed.

Joint work with Youzuo Lin (Arizona State University), Hongbin Guo (Arizona State University)

Image restoration by Tikhonov regularization based on generalized Krylov subspace methods

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Wed 11:00, Room Galilei

We describe Tikhonov regularization of large linear discrete ill-posed problems with a regularization operator of general form and present an iterative scheme based on a generalized Krylov subspace method. This method simultaneously reduces both the matrix of the linear discrete ill-posed problem and the regularization operator. The reduced problem so obtained may be solved, e.g., with the aid of the singular value decomposition. Also, multiparameter Tikhonov regularization is discussed. Numerical results illustrate the promise of problem-oriented operator in image denoising and deblurring.

Joint work with L. Reichel (Kent State University) and Q. YE (University of Kentucky)