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## Structured Matrices

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### Fifteen years of structured matrices

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 Tue 11:00, Room Pacinotti

The expression *structured matrices* appeared for the first time in a conference title in 1995, specifically in the session “Algorithms for Structured Matrices”, organized within the SPIE conference, held in San Diego (USA) [1, Session 6] and in the “Minisymposium on Structured Matrices” within the ILAS conference, held in Atlanta (USA) [2]. These first experiences led to the organization, in 1996, of the first two conferences specifically devoted to structured matrices: “International Workshop on Numerical Methods for Structured Matrices in Filtering and Control”, held in Santa Barbara (USA) [3] and “Toeplitz Matrices: Structures, Algorithms and Applications” held in Cortona (Italy) [4].

The organization of specific conferences on structured matrices has given the opportunity to meet together researchers working on theoretical and computational properties of structured matrices, and researchers working on applications. This exchange of experts from different fields has led to strong benefits to researches interested in structured matrices.

This anniversary gives the opportunity to reflect on the state-of-art in the research involving matrix structures in the last 15 years. The aim of this talk is to survey some key results achieved along different directions, paying special attention to the significant contribution of the Italian research group on structured numerical linear algebra, having its main site in Pisa. In particular, the four editions of the Cortona Workshop offer a privileged point of view in this context. Some pointers to future research perspectives will be also given.

- [1] Advanced Signal Processing Algorithms, SPIE Vol. 2563, pp. 266–313, 1995.
- [2] Proceeding of the Fifth Conference of the International Linear Algebra Society, Atlanta, Georgia (1995). Linear Algebra Appl. Vol. 254, pp. 1–5, 1997.
- [3] <http://www-control.eng.cam.ac.uk/extras/conferences/WNMSMFC96>
- [4] Calcolo Vol. 33, pp. 1–10, 1996.

Joint work with S. Serra Capizzano (Università dell’Insubria)

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### An enhanced plane search scheme for complex-valued tensor decompositions

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 Tue 11:50, Room Pacinotti

A third-order tensor  $T_1 \in \mathbb{C}^{l \times m \times n}$  is rank one if it can be written as the outer product of three nonzero vectors, i.e.,  $T_1 = a_1 \circ b_1 \circ c_1$ . The CANDECOMP/PARAFAC (CP) decomposition writes a given tensor  $T$  as a sum of  $R$  rank-one tensors  $T_i = a_i \circ b_i \circ c_i$ . Factor matrices  $A$ ,  $B$  and  $C$  are obtained by stacking the component vectors, e.g.,  $A = (a_1 a_2 \dots a_R)$ .

The most popular methods for computing CP are of the alternating least squares type (ALS). These methods have many drawbacks: they can take many iterations to converge, they are not guaranteed to converge to a global minimum or even a stationary point, and the final solution can heavily depend on the starting value. Moreover, these algorithms ignore structure of the given tensor, such as symmetry.

Recently, an enhanced line search (ELS) procedure has been proposed for improving ALS. In ELS a system of polynomial equations in two variables needs to be solved in each iteration. This subproblem can be much more expensive than the initial ALS iteration.

We propose an enhanced plane search (EPS) as an alternative to ELS. The corresponding polynomial subproblem is much easier to solve. We combine EPS with the single-step least squares algorithm (SSLS) that has recently been proposed for the computation of a CP with factors  $A, B, C$  such that  $A = B$  and  $A$  proportional to  $\bar{C}$ . The original SSLS algorithm is very cheap, but its convergence is not guaranteed. Our algorithm always converges and has better performance.

Joint work with L. De Lathauwer (K.U.Leuven: Campus Kortrijk and E.E. Dept. (ESAT), Belgium)

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### Structured perturbation theory of LDU factorization and accurate computations for diagonally dominant matrices

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 Tue 17:35, Room Pacinotti

If an LDU factorization with well-conditioned  $L$  and  $U$  factors of a given matrix  $A$  can be accurately computed, then the SVD of  $A$  can also be accurately computed [1] and the system of equations  $Ax = b$  can be accurately solved for almost all right hand sides [2], independently of the magnitude of the traditional condition number of  $A$ . These facts have motivated in the last years the development of *structured* algorithms for computing accurate LDU factorizations of structured matrices with well-conditioned  $L$  and  $U$  factors. One of the most important classes of matrices arising in applications is the class of *diagonally dominant matrices*, and recently an *structured* algorithm for computing their LDU factorizations has been introduced in [3]. Unfortunately, the best error bound proven in [3] for this algorithm is  $6n8^{(n-1)}\epsilon$ , where  $n$  is the size of the matrix and  $\epsilon$  is the unit roundoff. This bound is completely useless for sizes as small as  $n = 20$  in double precision. We present in this talk a new structured perturbation theory for the LDU factorization of diagonally dominant matrices parameterized in a certain way, that allows us to prove an error bound  $14n^3\epsilon$  for the LDU factorization computed with the algorithm in [3]. These results guarantee accurate computations of SVD and system solutions for any diagonally dominant matrix.

- [1] J. Demmel, M. Gu, S. Eisenstat, I. Slapničar, K. Veselić, and Z. Drmač, *Computing the singular value decomposition with high relative accuracy*, Linear Algebra Appl. **299**(1–3), 21–80 (1999)
- [2] F. M. Dopico and J. M. Molera, *Accurate solutions of structured linear systems*, in preparation.
- [3] Q. Ye, *Computing singular values of diagonally dominant matrices to high relative accuracy*, Math. Comp. **77**(264), 2195–2230 (2008)

Joint work with Plamen Koev (San Jose State University, CA, USA)

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### Using quasiseparable structure for polynomial roots computations

Y. EIDELMAN, Tel Aviv University, Israel  
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Wed 12:40, Room Pacinotti

The effective tool to compute all the roots of a polynomial is to determine the eigenvalues of the corresponding companion matrix using the QR iteration method. It turns out that the companion matrix belongs to a class of structured matrices which is invariant under QR iterations. Every matrix in this class has quasiseparable structure. This structure may be used to develop fast algorithms to compute eigenvalues of companion matrices. We discuss implicit fast QR eigenvalue algorithms solving this problem. The obtained algorithm is of complexity  $O(N^2)$  in contrast to  $O(N^3)$  for non-structured methods. The presentation is mainly based on the results of papers [1] and [2].

[1] S. Chandrasekaran, M. Gu, J. Xia, J. Zhu, A fast QR Algorithm for Companion Matrices, Operator Theory: Advances and Applications. 179, pp 111-143, 2007.

[2] D. A. Bini, P. Boito, Y. Eidelman, L. Gemignani, I. Gohberg, A Fast Implicit QR Algorithm for Companion Matrices, LAA. 432, 8, pp. 2006-2031, 2010.

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### Rank-structured matrix technology for solving nonlinear equations

L. GEMIGNANI, University of Pisa, Italy  
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Wed 11:25, Room Pacinotti

In this talk we discuss the use of rank-structured matrix methods for solving certain nonlinear equations arising in applications.

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### Inverses of pentadiagonal recursion matrices

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Tue 16:45, Room Pacinotti

Schemes for approximating matrix functions of the form  $f(A)v$ , where  $A$  is a large, possibly sparse, symmetric matrix, based on projections onto the extended Krylov subspace are currently being explored. Short recursion relations for generating orthonormal bases of extended Krylov subspaces of the type  $\mathbb{K}^{m,mi+1}(A) = \text{span}\{A^{-m+1}v, \dots, A^{-1}v, v, Av, \dots, A^m v\}$ ,  $m = 1, 2, 3, \dots$ , with  $i$  a positive integer have been developed. The recursion matrix associated with these recursion relations is pentadiagonal. The inverse of the recursion matrix associated with  $i = 2$  is also pentadiagonal. This structure does not necessarily hold for  $i > 2$  but a bandwidth structure for the inverse is maintained where the bandwidth increases with an increase in  $i$ . We discuss the structure of these inverses and present an application to the computation of rational Gauss quadrature rules.

Joint work with Lothar Reichel (Kent State University, Kent, OH 44242, USA)

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### A fast algorithm for updating and downsizing the dominant kernel principal components

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Tue 11:25, Room Pacinotti

Many important kernel methods in the machine learning area, such as kernel principal component analysis, feature approximation, denoising, compression and prediction require the computation of the dominant set of eigenvectors of the symmetric kernel Gram matrix. Recently, an efficient incremental approach was presented for the fast calculation of the dominant kernel eigenbasis [1], [2]. In this talk we propose faster algorithms for incrementally updating and downsizing the dominant kernel eigenbasis. These methods are well-suited for large scale problems since they are both efficient in terms of complexity and data management.

[1] G. Gins, I. Y. Smets, and J. F. Van Impe, Efficient tracking of the dominant eigenspace of a normalized kernel matrix, Neural. Comput., 20, (2008), pp. 523–554.

[2] L. Hoegaerts, L. De Lathauwer, I. Goethals, J. A. K. Suykens, J. Vandewalle, and B. De Moor, Efficiently updating and tracking the dominant kernel principal components, Neural Network., 20, (2007), pp. 220–229.

Joint work with Eugene E. Tyrtshnikov (Russian Academy of Sciences, Moscow, Russia), P. Van Dooren (Catholic University of Louvain, Louvain-la-Neuve, Belgium)

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### Generalized circulant preconditioners for Toeplitz systems

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Tue 15:25, Room Pacinotti

A Toeplitz matrix with the first entry of each column obtained by multiplying the last entry of the preceding column by  $e^{i\phi}$  is said to be an  $\{e^{i\phi}\}$ -circulant matrix. In this talk a formula for the distance in the Frobenius norm of a Toeplitz matrix to the set of the  $\{e^{i\phi}\}$ -circulant matrices is presented. Since the underlying minimization problem generalizes a minimization problem solved by the optimal circulant preconditioner introduced by T. Chan, the natural application of generalized circulants as preconditioners in the PCG method for solving linear systems with a Toeplitz matrix  $T_n$  is discussed. Similarly to the circulant case, matrix-vector products  $C_n y$  and  $C_n^{-1} y$ , where  $C_n$  is an  $\{e^{i\phi}\}$ -circulant and  $y$  any vector in  $\mathbb{C}^n$ , can be evaluated in  $\mathcal{O}(n \log n)$  arithmetic floating-point operations with the aid of the Fast Fourier Transform. Moreover, the construction of these generalized preconditioners does not require the explicit knowledge of the generating function of  $T_n$ , and needs only  $\mathcal{O}(n)$  operations. I present theoretical and numerical results that shed light on the performance of these preconditioners. Extensions to generalized circulant preconditioners for two-level Toeplitz systems are also discussed.

Joint work with L. Reichel (Kent State University)

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### On the power of randomized preconditioning

V. Y. PAN, Lehman College, CUNY, USA  
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Wed 12:15, Room Pacinotti

Our randomization techniques turn out to be an important missing ingredient of the known methods of preconditioning and thus dramatically expand their power. For a typical ill conditioned input we perform with a high precision only a small fraction of all flops involved, thus yielding dramatic acceleration of the known algorithms for general and structured input matrices, both in terms of the bit-operation count and

the CPU time observed. For Hankel and Toeplitz linear systems of  $n$  equations we save the factor  $a(n)$  where  $a(512) > 15$ ,  $a(1024) > 90$ , and  $a(2048) > 350$ . Our work extends the domain of application for iterative refinement of the solution of a linear system of equations and for Newtons iteration for the inversion of general and structured matrices and enables effective treatment of nearly rank deficient (e.g., nearly singular) general and structured matrices with no pivoting and no orthogonalization. Such matrices regularly appear, e.g., in the Inverse Iteration for eigen-solving, and our approach enables us to incorporate iterative refinement without slowing down the convergence. Further extensions include polynomial root-finding, computation of the numerical rank of a matrix, approximation of a nearly rank deficient matrix with a matrix of a smaller rank, and approximation of a matrix by a nearby Toeplitzlike or Hankel-like matrix. Our 30-minute talk shall outline this 5-year work by presenting the main techniques and some central formal and experimental results, with the pointers to more complete and detailed coverage in our papers in LAA 2009/2010, in press, and in progress.

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### Designing a library for structured linear algebra computation.

G. RODRIGUEZ, University of Cagliari, Italy  
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 Tue 15:00, Room Pacinotti

Two libraries recently developed for computation with structured matrices will be presented. The plans for extending the functionality of these libraries and for integrating them with other available software will be described.

- [1] A. Aricò and G. Rodriguez. A fast solver for linear systems with displacement structure. Manuscript.
- [2] M. Redivo-Zaglia and G. Rodriguez. `smt`: a Matlab structured matrices toolbox. arXiv:0903.5094 [math.NA], 2009. Submitted.

Joint work with A. Aricò and F. Arrai (University of Cagliari), M. Redivo-Zaglia (University of Padova).

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### Error Analysis of a Fast Algorithm for Quasiseparable Systems

MICHAEL STEWART, Georgia State University  
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 Wed 11:00, Room Pacinotti

This work describes a parameterization of an  $n \times n$  quasiseparable matrix  $A$  in terms of a nested product of small Householder transformations and very sparse bidiagonals. Once computed the parameterization can be exploited for fast,  $O(n)$ , solution of systems of equations with quasiseparable structure. The representation is insensitive in the sense that small errors on the parameters correspond to small errors on the matrix. Results of an error analysis show that the algorithm is normwise backward stable.

Joint work with Tom Bella (University of Rhode Island) and Vadim Olshevsky (University of Connecticut)

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### On structure-preserving Arnoldi-like methods

A. SALAM, University Lille Nord de France, France.  
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 Tue 15:50, Room Pacinotti

In this talk, two structure-preserving Arnoldi-like methods are presented and studied. The obtained methods preserve the

structures of a class of structured matrices, including Hamiltonian, skew-Hamiltonian or symplectic matrices. Such methods are useful for computing few eigenvalues and vectors of large and sparse structured matrices. Numerical experiments are given.

- [1] A. Bunse-Gerstner and V. Mehrmann, A symplectic QR-like algorithm for the solution of the real algebraic Riccati equation, IEEE Trans. Automat. Control AC-31 (1986), 1104–1113.
- [2] A. Salam and E. Al-Aidarous and A. Elfarouk, Optimal symplectic Householder transformations for  $SR$ -decomposition, Linear Algebra Appl. 429 (2008), no. 5-6, 1334-1353.
- [3] M. Sadkane and A. Salam, A note on symplectic block reflectors, ETNA, Vol. 33 (2009), pp 45-52.

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### Unmixing of rational functions by tensor computations

M. VAN BAREL, Katholieke Universiteit Leuven, Belgium  
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 Tue 12:15, Room Pacinotti

Very recently, Lieven De Lathauwer has introduced various types of Block Term Decompositions (BTD) for higher-order tensors. These decompositions generalize both the Tucker decomposition (or multilinear Singular Value Decomposition) and the Canonical / Parallel Factor decomposition. The latter are related with low multilinear rank approximation and low rank approximation of higher-order tensors, respectively. It turns out that BTDs can be used for source separation and factor analysis. In this talk, we investigate the possibility of unmixing rational functions using a particular type of BTD. We show the effectiveness of using the BTD by some numerical examples.

Joint work with L. De Lathauwer (Katholieke Universiteit Leuven, Belgium)

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### A multishift QR-algorithm for hermitian plus low rank matrices

R. VANDEBRIL, K.U.Leuven, Belgium  
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 Wed 11:50, Room Pacinotti

Hermitian plus possibly non-Hermitian low rank matrices can be efficiently reduced into Hessenberg form. The resulting Hessenberg matrix can still be written as the sum of a Hermitian plus low rank matrix.

In this talk we will discuss a new implicit multishift  $QR$ -algorithm for Hessenberg matrices, which are the sum of a Hermitian plus a possibly non-Hermitian low rank correction.

The proposed algorithm exploits both the symmetry and low rank structure to obtain a  $QR$ -step involving only  $O(n)$  floating point operations instead of the standard  $O(n^2)$  operations needed for performing a  $QR$ -step on a Hessenberg matrix. The algorithm is based on a suitable  $O(n)$  representation of the Hessenberg matrix. The low rank parts present in both the Hermitian and low rank part of the sum are compactly stored by a sequence of Givens transformations and few vectors.

Due to the new representation, we cannot apply classical deflation techniques for Hessenberg matrices. A new, efficient technique is developed to overcome this problem.

Some numerical experiments based on matrices arising in applications are performed. The experiments illustrate effectiveness and accuracy of both the  $QR$ -algorithm and the newly developed deflation technique.

Joint work with G. M. Del Corso (University of Pisa)

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**Twisted Green's (CMV-like) matrices and their factorizations, Laurent polynomials and Digital Filter Structures.**

P. ZHLOBICH, University of Connecticut, USA

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*Tue 17:10, Room Pacinotti*

Several new classes of structured matrices have appeared recently in the scientific literature. Among them there are so-called CMV and Fiedler matrices which are found to be related to polynomials orthogonal on the unit circle and Horner polynomials, respectively. Both matrices are five diagonal and have a similar structure, although they have appeared under completely different circumstances.

In a recent paper by Bella, Olshevsky and Zhlobich, it was proposed a unified approach to the above mentioned matrices. Namely, it was shown that all of them belong to a wider class of twisted Green's matrices. We will use this idea to show that the factorizability of CMV and Fiedler matrices into a product of planar rotations in the  $n$ -dimensional space is also inherited by twisted Green's matrices. Shortly, for a given Hessenberg Green's matrix of size  $n$ , the interchange of factors in the factorization leads to  $2^n$  different twisted Green's matrices.

CMV matrix appeared in the scientific literature in connection with Laurent polynomials orthogonal on the unit circle. Fiedler matrix was developed purely from its factorization. We will show that an infinite-dimensional twisted Green's matrix serve as the operator of multiplication by "z" in the linear space of complex Laurent polynomials. Our development doesn't use orthogonality in any sense and is based on the factorization and recurrence relations only. In the case of finite dimensional matrices we are able to give an explicit form of an eigenvector and all the generalized eigenvectors for a given eigenvalue.

The final part of our talk will be devoted to Kimura's approach to CMV matrices, i.e. Signal Flow Graphs (SFG) approach. We will exploit the tool of SFG to visualize all the theoretical results for twisted Green's matrices as well as to show how they can be used in construction of new types of Digital Filters.

Joint work with F. Marcellán (Univ. Carlos III de Madrid), V. Olshevsky (Univ. of Connecticut) and G. Strang (MIT)

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