$$\begin{cases} u' = -\frac{3}{4}u - \frac{5}{4}v - \frac{\sqrt{3}}{2}w \\ v' = -\frac{5}{4}u - \frac{3}{4}v + \frac{\sqrt{3}}{2}w \\ w' = \frac{\sqrt{3}}{4}u - \frac{\sqrt{3}}{4}v + \frac{1}{2}w \end{cases}$$

Sistema del primo ordine in $\mathbb{R}^3$ – OMOGENEO

$$Y' = AY \qquad A = \begin{pmatrix} -\frac{3}{4} & -\frac{5}{4} & -\frac{\sqrt{3}}{2} \\ -\frac{5}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4} & \frac{1}{2} \end{pmatrix}$$

$$Y = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Cerco gli autovalori / autovettori di A.

$$\det\begin{pmatrix} -\frac{3}{4}-\lambda, & -\frac{5}{4}, & -\frac{\sqrt{3}}{2} \\ -\frac{5}{4}, & -\frac{3}{4}-\lambda, & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{4}, & -\frac{\sqrt{3}}{4}, & \frac{1}{2}-\lambda \end{pmatrix} = \left(-\frac{3}{4}-\lambda\right)\left(-\frac{3}{4}-\lambda\right)\left(\frac{1}{2}-\lambda\right) + \left(-\frac{5}{4}\right)\frac{\sqrt{3}}{2}\,\frac{\sqrt{3}}{4}$$

$$+\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{5}{4}\right)\left(-\frac{\sqrt{3}}{4}\right) - \left(-\frac{3}{4}-\lambda\right)\left(-\frac{\sqrt{3}}{4}\right)\frac{\sqrt{3}}{2}$$

$$-\left(-\frac{5}{4}\right)\left(-\frac{5}{4}\right)\left(\frac{1}{2}-\lambda\right) - \frac{\sqrt{3}}{4}\left(-\frac{3}{4}-\lambda\right)\left(-\frac{\sqrt{3}}{2}\right)$$

$$\left(\frac{9}{16}+\frac{3}{2}\lambda+\lambda^2\right)\left(\frac{1}{2}-\lambda\right)-\frac{15}{32}-\frac{15}{32}-\left(\frac{3}{4}+\lambda\right)\frac{3}{8}-\frac{25}{16}\left(\frac{1}{2}-\lambda\right)$$

$$-\frac{3}{8}\left(\frac{3}{4}+\lambda\right)=$$

$$\frac{9}{32}+\frac{3}{4}\lambda+\frac{\lambda^2}{2}-\frac{9}{16}\lambda-\frac{3}{2}\lambda^2-\lambda^3-\frac{30}{32}-\frac{9}{32}-\frac{3}{8}\lambda-\frac{25}{32}+\frac{25}{16}\lambda$$

$$-\frac{9}{32}-\frac{3}{8}\lambda=$$

$$\boxed{-\lambda^3-\lambda^2+\lambda-2}=P(\lambda)$$

$$\frac{\cancel{9}-30-\cancel{9}-25-9}{32}=-\frac{64}{32}$$

si vede (!!?) che -2 è radice

$8-4-2-2=0$

DIVIDO $-\lambda^3-\lambda^2+\lambda-2$ per $\lambda+2$

$$\begin{array}{l|l}
-\lambda^3-\lambda^2+\lambda-2 & \lambda+2 \\ \hline
-\lambda^3-2\lambda^2 & -\lambda^2+\lambda-1 \\ \hline
\quad\ \ \lambda^2+\lambda-2 & \\
\quad\ \ \lambda^2+2\lambda & \\ \hline
\quad\quad\ -\lambda-2 &
\end{array}$$

$$\Rightarrow\quad(-\lambda^2+\lambda-1)(\lambda+2)=P(\lambda)$$

RADICI $= \underset{\lambda_1}{-2}, \quad \underset{\lambda_{23}}{\dfrac{1 \pm \sqrt{3}\, i}{2}}$ $\qquad \lambda^2 - \lambda + 1 = 0$

$\Rightarrow$ A ha tre autovalori in $\mathbb{C}^3$

$\vec{e}_1 \in \mathbb{R}^3 \quad \vec{w}_{1,2} = \underbrace{\vec{e}_2 \pm i\vec{e}_3}_{\in \mathbb{C}^3} \qquad e_2, e_3 \in \mathbb{R}^3$

Nella base di e_1, w_1, w_2 la matrice diventa

$$D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \frac{1}{2} + \frac{\sqrt{3}}{2} i & 0 \\ 0 & 0 & \frac{1}{2} - \frac{\sqrt{3}}{2} i \end{pmatrix}$$

IN QUESTO SISTEMA DI RIFERIMENTO IL SISTEMA DIVENTA

$$Z' = D Z \longleftrightarrow Z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

$$z_1' = -2 z_1, \quad z_2' = \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) z_2, \quad z_3' = \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) z_3$$

$$\Rightarrow \quad Z(x) = \begin{pmatrix} d_1 \, e^{-2x} \\ d_2 \, e^{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)x} \\ d_3 \, e^{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)ix} \end{pmatrix} =$$

$$d_1 e^{-2x} \vec{e}_1 + d_2 e^{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)x} \vec{w}_1 + d_3 e^{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)x} \vec{w}_2 =$$

$$(d_1, d_2, d_3 \in \mathbb{C})$$

$$d_1 e^{-2x} \vec{e}_1 + d_2 e^{\frac{x}{2}} e^{\frac{\sqrt{3}}{2}ix} \left(\vec{e}_2 + i \vec{e}_3\right)$$

$$+ d_3 e^{\frac{x}{2}} e^{-\frac{\sqrt{3}}{2}ix} \left(\vec{e}_2 - i \vec{e}_3\right) =$$

$$d_1 e^{-2x} \vec{e}_1 + e^{\frac{x}{2}} \left(d_2 e^{\frac{\sqrt{3}}{2}ix} + d_3 e^{-\frac{\sqrt{3}}{2}ix}\right) \vec{e}_2 +$$

$$+ e^{\frac{x}{2}} \left(i d_2 e^{\frac{\sqrt{3}}{2}ix} - i d_3 e^{-\frac{\sqrt{3}}{2}ix}\right) \vec{e}_3 =$$

$$d_1 e^{-2x} \vec{e}_1 + e^{\frac{x}{2}} \left(\underbrace{(d_2 + d_3)}_{c_2} \cos\left(\frac{\sqrt{3}}{2}x\right) + i \underbrace{(d_2 - d_3)}_{c_3} \sin\left(\frac{\sqrt{3}}{2}x\right) \right) \vec{e}_2$$

$$+ e^{\frac{x}{2}} \left(i (d_2 - d_3) \cos\left(\frac{\sqrt{3}}{2}x\right) - (d_2 + d_3) \sin\left(\frac{\sqrt{3}}{2}x\right) \right) \vec{e}_3$$

$$\boxed{c_1 e^{-2x}\vec{e}_1 + e^{\frac{x}{2}}\left(c_2\cos\left(\frac{\sqrt3}{2}x\right)+c_3\sin\left(\frac{\sqrt3}{2}x\right)\right)\vec{e}_2 + e^{\frac{x}{2}}\left(c_3\cos\left(\frac{\sqrt3}{2}x\right)-c_2\sin\left(\frac{\sqrt3}{2}x\right)\right)\vec{e}_3} = \circledast$$

Se c_1, c_2, c_3 variano in $\mathbb{R}$ ho trovato le sol. reali (in $\mathbb{R}^3$)

Se "parto" da un punto sulla retta $\{c\,\vec{e}_1\}$ rimango in questa retta (e mi muovo come $c_0 e^{-2x}\vec{e}_1$

Se parto nel piano $\{c_1\vec{e}_2 + c_2\vec{e}_3\}$, rimango nel piano.

<u>NOTA</u> che la sol $\circledast$ si può scrivere

$$\boxed{c_1 e^{-2x}\vec{e}_1 + e^{x/2} A\cos\left(\frac{\sqrt3}{2}x-\theta\right)\vec{e}_2 + e^{x/2} A\sin\left(\frac{\sqrt3}{2}-\theta\right)\vec{e}_3}$$

Perché $\exists$ $c_2\cos(t)+c_3\sin(t) = A\cos(t-\theta)$ (verificare!!)

$\Rightarrow$ $c_3\cos(t)-c_2\sin(t) = A\sin(t-\theta)$

SI PUÒ VEDERE CHE

$$e_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad e_3 = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$$

-2

$$w_{12} = \begin{pmatrix} 1 \\ -1 \\ I\iota \end{pmatrix}$$