

Calcolo di integrali

Si calcolino i seguenti integrali indefiniti
(= primitive delle funzioni integrande)

1. $\int \frac{1}{x \ln(x)^k} dx$
2. $\int \frac{\sin(x) + \cos(x)}{\cos(x)^3} dx$
3. $\int \sin(x)^4 dx$
4. $\int \tan(x)^3 dx$
5. $\int \tan(x)^4 dx$
6. $\int x e^{x^2} dx$
7. $\int e^{\sqrt{x}} dx$
8. $\int e^x \sin(x) dx$
9. $\int x^2 \operatorname{arctg}(x) dx$
10. $\int \operatorname{arctg}(\sqrt{x}) dx$
11. $\int \sin(\ln(x)) dx$
12. $\int \frac{1}{1 + \sin(x) + \cos(x)} dx$
13. $\int \frac{\sqrt{x}(x+2)}{x-1} dx$
14. $\int \frac{\sqrt{x}}{(x+2)\sqrt{x-1}} dx$
15. $\int \frac{\cos(x)}{1 + \cos(x)} dx$
16. $\int \frac{1}{\sqrt{1+x^2+x}} dx$
17. $\int \frac{x^2}{\sqrt{x^2-x}} dx$
18. $\int \frac{1}{x^2} \sqrt{\frac{x-1}{x+1}} dx$
19. $\int \frac{1}{x^3(1+x^2)} dx$
20. $\int \frac{x^2}{x^4+1} dx$
21. $\int \frac{x^2}{(1+x^2)^3} dx$
22. $\int \frac{x^3}{x^2-5x+6} dx$
23. $\int \frac{x+2}{x^3+1} dx$
24. $\int \frac{1}{(x^3-1)^2} dx$
25. $\int \frac{x+1}{(x^2+x+1)(2x+1)} dx$
26. $\int \frac{x}{(x^2+x+1)^2} dx$
27. $\int \ln(1+x^2) dx$
28. $\int \frac{x}{\sqrt{x+1}-\sqrt{x-1}} dx$
29. $\int x^2 2^{x+1} dx$
30. $\int x \operatorname{arctg}(\sqrt{x}) dx$
31. $\int \frac{\operatorname{arctg}(x)}{\sqrt{x}} dx$
32. $\int x \sin \sqrt{x-1} dx$
33. $\int \frac{x e^x}{(1+x)^2} dx$
34. $\int \frac{\ln(x)-1}{\ln(x)^2} dx$

$$35. \int \cos(5x) \sin(2x) dx$$

$$36. \int \ln(1 - \sqrt{x}) dx$$

$$37. \int \frac{1}{1 + \sqrt[3]{x}} dx$$

$$38. \int x \sqrt[3]{x-1} dx$$

$$39. \int \frac{1 + \sin(x)}{1 - \sin(x)} dx$$

$$40. \int \frac{\cos^2(x)}{1 - 4\sin^2(x)} dx$$

$$41. \int \frac{3 + \tan(x)}{\cos(x)} dx$$

$$42. \int \frac{1}{\cos(x)} dx$$

$$43. \int \frac{1}{1 + \cos(x)} dx$$

$$44. \int \frac{1 + 2\sin^2(x)}{1 + 2\cos^2(x)} dx$$

$$45. \int \frac{2 + \sqrt{x}}{x + x\sqrt{x}} dx$$

$$46. \int \frac{2 + \sqrt[3]{x}}{1 + \sqrt{x}} dx$$

Risposte

$$1. \frac{\ln(x)^{1-k}}{1-k} + \text{const.}, \text{ se } k \neq 1, \\ \ln(\ln|x|) + \text{const. se } k = 1$$

$$2. \frac{1}{2} \tan(x)^2 + \tan(x) + \text{const.} \\ (t = \tan(x))$$

$$3. \frac{3x}{8} - \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + \text{const.} \\ (\text{per parti ci si riconduce a} \\ \int \cos^2(x) dx)$$

$$4. \frac{1}{2} \tan(x)^2 + \ln(\cos(x)) + \text{const.} \\ (t = \tan(x))$$

$$5. \left(\frac{4}{3} + \frac{1}{3} \frac{1}{\cos(x)^2} \right) \tan(x) + \text{const.} \\ (t = \tan(x))$$

$$6. \frac{1}{2} e^{x^2} + \text{const.} \\ (t = x^2)$$

$$7. 2e^{\sqrt{x}}(\sqrt{x} - 1) + \text{const.} \\ (t = \sqrt{x} \text{ poi per parti})$$

$$8. \frac{1}{2} e^x(\sin(x) + \cos(x)) + \text{const.} \\ (\text{per parti})$$

$$9. -\frac{x^2}{6} + \frac{x^3 \arctan(x)}{3} + \frac{\ln(1+x^2)}{6} + \\ \text{const.} \\ (\text{per parti, poi Hermite})$$

$$10. -\sqrt{x} + (1+x) \arctan(\sqrt{x}) + \text{const.} \\ (t = \sqrt{x} \text{ poi per parti})$$

$$11. \frac{x}{2} (\sin(\ln(x)) - \cos(\ln(x))) + \text{const.} \\ (t = \ln(x) \text{ poi per parti})$$

$$12. \ln\left(1 + \tan\left(\frac{x}{2}\right)\right) + \text{const.} \\ (t = \tan(x/2))$$

$$13. 6\sqrt{x} + \frac{2\sqrt{x^3}}{3} + 3 \ln\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \text{const.} \\ (t = \sqrt{x}, \text{ poi Hermite})$$

$$14. \sqrt{x(x-1)} - 5 \ln(\sqrt{x} + \sqrt{x-1}) + \\ + \sqrt{\frac{3}{2}} \ln\left(\frac{-2 + 2\sqrt{6}\sqrt{x(x-1)} + 5x}{2 + 2\sqrt{6}\sqrt{x(x-1)} - 5x}\right) + \\ \text{const.} \\ (t = \sqrt{\frac{x}{x-1}}, \text{ poi Hermite})$$

$$15. x - \tan(x/2) + \text{const.} (t = \tan(x/2))$$

16. $-\frac{x^2}{2} + \frac{x\sqrt{1+x^2}}{2} + \frac{1}{2} \arcsin(x) + const.$
($\sqrt{1+x^2} = x+t$, poi Hermite)
17. $\frac{\sqrt{x}(2x^2+x-3)}{4\sqrt{x-1}} + \frac{3}{4} \ln(\sqrt{x-1} - \sqrt{x}) + const.$
($\sqrt{x^2-x} = x+t$ oppure $t = \sqrt{\frac{x-1}{x}}$, poi Hermite)
18. $-\frac{\sqrt{x^2-1}}{x} - \arctan\left(\frac{1}{x^2-1}\right) + const.$
($t = \sqrt{\frac{x-1}{x+1}}$, poi Hermite)
19. $-\frac{1}{2x^2} + \ln\left(\frac{1+x^2}{x}\right) + const.$
(Hermite)
20. $\frac{\sqrt{2}}{8} \ln(x^2 - \sqrt{2}x + 1) - \frac{\sqrt{2}}{8} \ln(x^2 + \sqrt{2}x + 1) + \frac{\sqrt{2}}{4} \operatorname{artg}(\sqrt{2}x + 1) + \frac{\sqrt{2}}{4} \operatorname{artg}(\sqrt{2}x - 1) + const.$
(Hermite)
21. $\frac{1}{8} \operatorname{arctg}(x) + \frac{x}{8(1+x^2)} - \frac{x}{4(1+x^2)^2} + const.$
(Hermite)
22. $5x + \frac{1}{2}x^2 + 27 \ln(x-3) - 8 \ln(x-2) + const.$
(Hermite)
23. $\sqrt{3} \operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) +$
24. $\frac{1}{6} \ln\left(\frac{(1+x)^2}{1-x+x^2}\right) + const.$
(Hermite)
25. $-\frac{x}{3(x^3-1)} + \frac{2\sqrt{3}}{9} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{9} \ln\left(\frac{1+x+x^2}{(x-1)^2}\right) + const.$
(Hermite)
26. $\frac{\sqrt{3}}{3} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{6} \ln\left(\frac{(2x+1)^2}{1+x+x^2}\right) + const.$
(Hermite)
27. $-\frac{2\sqrt{3}}{9} \operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{2+x}{3(1+x+x^2)} + const.$
(Hermite)
28. $-2x + 2 \operatorname{arctg}(x) + x \ln(1+x^2) + const.$
(per parti)
29. $\frac{1}{15}(5x^2 - x - 2)\sqrt{x-1} + \frac{1}{15}(5x^2 + x - 2)\sqrt{x+1} + const.$
($t = \sqrt{\frac{x+1}{x-1}}$, poi Hermite)
30. $\frac{2^{x+1}}{\ln^3(2)} (\ln^2(2)x^2 - 2 \ln(2)x + 2) + const.$
(per parti)
31. $\frac{1}{2}(x^2-1) \operatorname{arctg}(\sqrt{x}) - \frac{1}{6} \sqrt{x}(x-3) + const.$
($t = \sqrt{x}$, poi per parti, poi Hermite)
32. $2\sqrt{x} \operatorname{arctg}(x) + \int \frac{4t^2}{1+t^4} dt$, dove $t = \sqrt{x}$
poi csi continua come in 20

32. $2\sqrt{x-1}(6-x)\cos(\sqrt{x-1}) + 2(3x-8)\sin(\sqrt{x-1}) + const.$
(si pone $t = \sqrt{x-1}$ e poi si integra per parti)
33. $\frac{e^x}{1+x} + const.$ // (usando in modo furbo l'integrazione per parti)
34. $\frac{x}{\ln(x)} + const.$ (si pone $t = \ln(x)$ poi si procede come nel 33)
35. $\frac{5}{21}\sin(5x)\sin(2x) + \frac{2}{21}\cos(5x)\cos(2x) + const.$
(per parti, due volte, poi furbizia)
36. $(x-2)\ln(1-\sqrt{x}) - 2\sqrt{x} - x - \frac{2}{3}\sqrt{x^3} - \frac{1}{2}x^2 + const.$
(prima per parti, poi $t = \sqrt{x}$ infine Hermite)
37. $-3\sqrt[3]{x} + \frac{3}{2}\sqrt[3]{x^2} + 3\ln(1-\sqrt[3]{x}) + const.$
($t = \sqrt[3]{x}$)
38. $\frac{\sqrt[3]{x-1}}{28}(12x^2 - 3x - 9) + const.$
($t = \sqrt[3]{x-1}$)
39. $-\frac{4}{\tan(x/2) - 1} - x + const.$
($t = \tan(x/2)$)
40. $\frac{x}{4} + \frac{\sqrt{3}}{4}\text{SettTanh}(\sqrt{3}\tan(x)) + const.$
($t = \tan(x)$)
41. $-\frac{2}{t^2-1} + 3\ln\left(\frac{t+1}{t-1}\right) + const.$
dove $t = \tan(x/2)$ (sostituzione $t = \tan(x/2)$ poi Hermite)
42. $\ln\left(\frac{\sin(x/2) + \cos(x/2)}{\sin(x/2) - \cos(x/2)}\right) + const.$
($t = \tan(x/2)$)
43. $\tan(x/2) + const.$ ($t = \tan(x/2)$)
44. $-5\tan(x) + \tan^3(x) + \frac{16}{\sqrt{3}}\text{arctg}\left(\frac{\tan(x)}{\sqrt{3}}\right) + const.$
($t = \tan(x)$)
45. $2\ln\left(\frac{x}{1+\sqrt{x}}\right) + const.$
($t = \sqrt{x}$)
46. $-3\sqrt[3]{x} + 4\sqrt{x} + \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{3}\text{arctg}\left(\frac{2\sqrt[6]{x}-1}{\sqrt{3}}\right) + \ln\left(\frac{1+\sqrt[6]{x}}{(1+\sqrt{x})^3}\right) + const.$
($t = \sqrt[6]{x}$)