[1]

1. $f(t)$

no	sì	sì
0	1	∞

per $t \to 0$ $\quad f(t) \sim -\frac{\lg t}{t} > \frac{1}{t}$

per $t \to 1$ $\quad f(t) \to 1$ disc. eliminabile

per $t \to +\infty$ $\quad f(t) \sim \frac{\lg t}{t^2} < \left(\frac{1}{t}\right)^{2-\varepsilon}$

$F(x)$

CE $\quad x > 0$

SGN $\quad f(t) > 0$; $x < \sqrt{x}$ se $0 < x < 1$

+	−
0	1

LIM $\quad$ per $x \to 0$ $\quad F(x) = \frac{(\sqrt{x} - x)\,\lg \xi}{\xi(\xi - 1)}$ $\quad$ con $x < \xi < \sqrt{x}$

$$\sim \frac{\sqrt{x}\,|\lg \xi|}{\xi} > \frac{\sqrt{x}\,|\lg \sqrt{x}|}{\sqrt{x}} \to +\infty$$

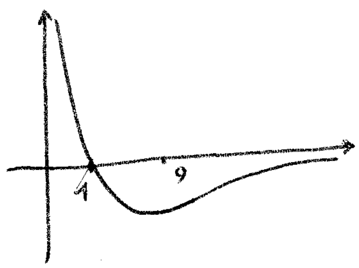
per $x \to +\infty$ $\quad F(x) \to 0$

per $x \to 1$ $\quad F(x) \to 1$

DRV $\quad F'(x) = \frac{\lg x}{x(\sqrt{x} - 1)}\left(\frac{1}{4} - \frac{1}{\sqrt{x} + 1}\right)$, $\quad x > 0,\ x \neq 1$

$F'(x) \geq 0 \iff \sqrt{x} \geq 3 \iff x \geq 9$

per $x \to 1$ $\quad F'(x) \to -\frac{1}{2}$



2. La fz. è dispari in sen$x$. Si pone $\cos x = t$, $-\mathrm{sen}\,x\,dx = dt$.

$$\int \mathrm{sen}x \,\frac{1 + 2\cos x}{(1 - \cos^2 x)(3 - \cos x)}\,dx = -\int \frac{1 + 2t}{(t^2 - 1)(t - 3)}\,dt \underset{\text{Hermite}}{=}$$

$$= \int \left(\frac{3}{4}\,\frac{1}{t - 1} + \frac{1}{8}\,\frac{1}{t + 1} - \frac{7}{8}\,\frac{1}{t - 3}\right) dt = \cdots$$

3.

C.E. $\quad x, y \in \mathbb{R}$

$y = 0$ slz. costante

$\frac{1}{B(y)} = \frac{1}{\sqrt[3]{y}}$ è integrabile in un intorno di 0.

ci sono slz. che intersecano quella costante.

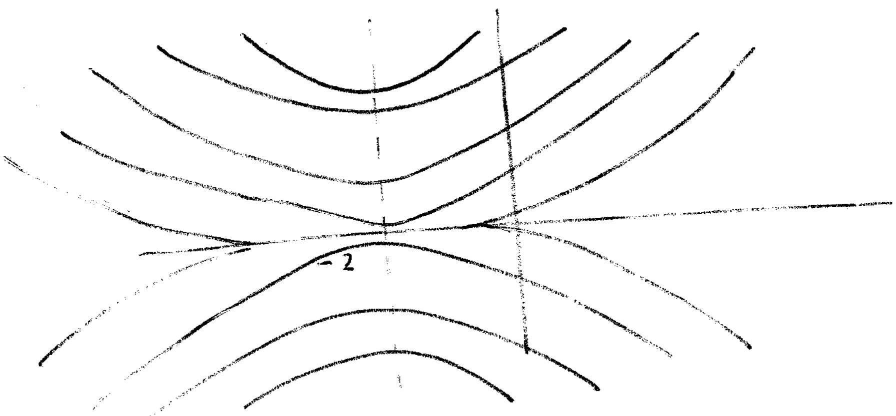
$$\int \frac{dy}{\sqrt[3]{y}} = \int \sqrt[3]{x+2}\, dx \quad \Rightarrow \quad \frac{3}{2} y^{2/3} = \frac{3}{4}(x+2)^{4/3} + \frac{3}{4}c$$

$\vdots$

$$y = \pm \sqrt{\left(\frac{(x+2)^{4/3} + c}{2}\right)^3}$$

$x \quad c > 0 \qquad x \in \mathbb{R}$

$x \quad c < 0 \qquad x > -2 + \sqrt[4]{-c^3}$ opp $x < -2 - \sqrt[4]{-c^3}$



4.

$x > 1$:

$a_n \to 0$

$$a_n = \frac{\sqrt{1+x^{2n}} - x^n}{\sqrt{1+x^{2n}}} = \frac{1}{\sqrt{1+x^{2n}}\left(\sqrt{1+x^{2n}} + x^n\right)} \sim$$

$$\sim \frac{1}{x^n \cdot 2x^n} = \frac{1}{2}\left(\frac{1}{x^2}\right)^n$$

termine generale di una serie geometrica conv.

$0 < x \leq 1$:

$a_n \not\to 0$

la serie diverge.

[2]

1. $f(t)$: sì (0) sì (1) no (∞)

- per $t \to 0$ $\quad f(t) \to 0$ $\quad$ D. ELIMINABILE
- per $t \to 1$ $\quad f(t) \to 1$ $\quad$ "
- per $t \to +\infty$ $\quad f(t) \sim \dfrac{t^2}{\lg t} \to +\infty$

$F(x)$:

C.E. $x \geq 0$

SGN $f(t) > 0$; $x < x^3$ se $x > 1$. Dunque $-$ (0, 1) $+$ (da 1)

LIM per $x \to 0$ e per $x \to 1$ $\quad F(x) \to 0$

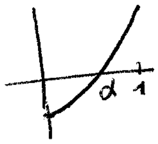
per $x \to +\infty$ $\quad F(x) = \dfrac{(x^3 - x)\,\xi(\xi - 1)}{\lg \xi} \sim \dfrac{x^3 \xi^2}{\lg \xi} > \dfrac{x^5}{3 \lg x} \to +\infty$ senza asintoto

$x < \xi < x^3$

DRV $F'(x) = \dfrac{x^5(x^3-1)}{\lg x} - \dfrac{x(x-1)}{\lg x}$

$= \dfrac{x(x-1)}{\lg x}\,(x^6 + x^5 + x^4 - 1)$

il rapporto è positivo, il termine in parentesi è una funzione crescente



per $x \to 0$ $\quad F'(x) \to 0$

per $x \to 1$ $\quad F'(x) \to 2$

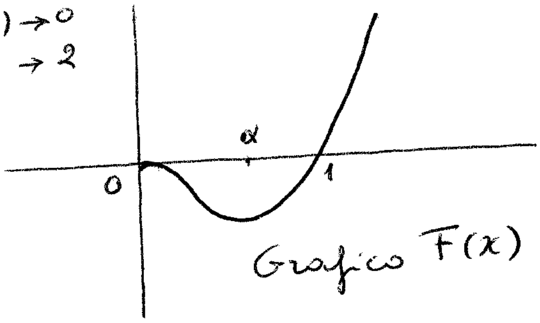


Grafico $F(x)$

2. Ponendo $\operatorname{tg} x = t$:

$$\int \frac{t^3 + 2t}{(t+4)(1+t^2)}\,dt \underset{\text{dividendo i due polinomi}}{=} 1 - \frac{4t^2 - t + 4}{(t+4)(t^2+1)} \underset{\text{Hermite}}{= \ldots} =$$

$$= \int \left(1 - \frac{72}{17}\,\frac{1}{t+4} + \frac{1}{17}\,\frac{4t+1}{t^2+1}\right) dt =$$

$$= t - \frac{72}{17} \lg|t+4| + \frac{2}{17} \lg(t^2+1) + \frac{1}{17} \operatorname{arctg} t + c = \ldots$$

3. C.E. $x, y \in \mathbb{R}$

$y = 0$ sol. costante

$1/B(y) = 1/\sqrt[3]{y}$ è integrabile nell'intorno di 0.

Ci sono soluzioni che intersecano la sol. costante.

$$\int \frac{dy}{\sqrt[3]{y}} = \int \sqrt[3]{(x-2)}\,dx$$

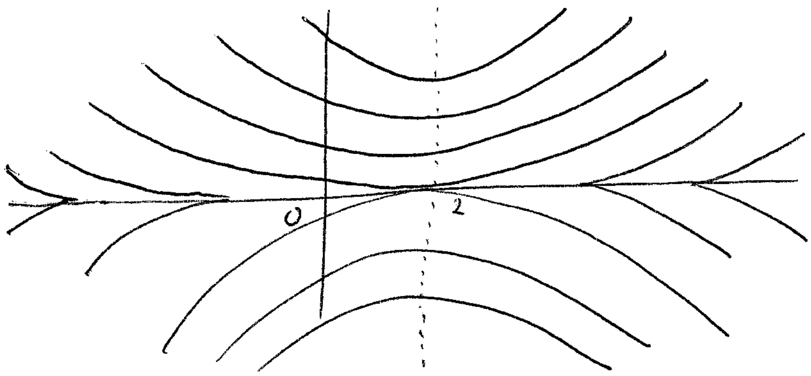
$\vdots$

$$y = \pm \sqrt{\left(\frac{(x-2)^{4/3} + c}{2}\right)^3}$$

Deve essere $(x-2)^{4/3} + c \geq 0$.

Se $c \geq 0$, è sempre verificato.

Se $c < 0$, deve essere $x > 2 + (-c)^{3/4}$ opp. $x < 2 - (-c)^{3/4}$.



4. Criterio rapporto:

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!^x}{(2n+2)!}\,\frac{(2n)!}{(n!)^x} = \frac{(n+1)^x}{(2n+1)\,2(n+1)} \cong \frac{n^x}{4n^2} =$$

$$= \frac{1}{4}\left(\frac{1}{n}\right)^{2-x}$$

$$\frac{a_{n+1}}{a_n} \to \begin{cases} 0 & \text{se } x < 2 \\ 1/4 & \text{se } x = 2 \\ +\infty & \text{se } x > 2 \end{cases}$$

La serie converge se $x \leq 2$.