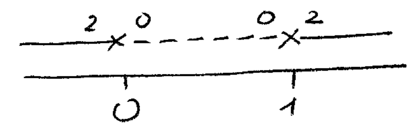
Soluzioni [1]

1. $\begin{cases} 2x^2-2x+1>0 & \to \text{sempre verificata} \\ |x|/\sqrt{2x^2-2x+1} \le 1 \iff x^2-2x+1 \ge 0 & \to \text{sempre verificata} \end{cases}$

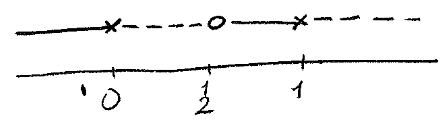
C.E. : $\mathbb{R}$

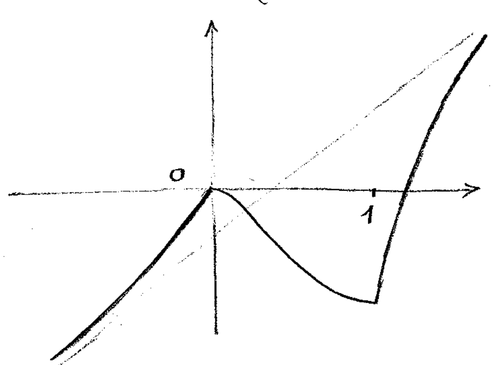
LIM. per $x \to \pm\infty$ $\quad f(x) \to \pm\infty \qquad y = x - \frac{\pi}{4}$ as. obliquo

DRV $\quad f'(x) = 1 - \dfrac{\operatorname{sgn} x \operatorname{sgn}(1-x)}{2x^2-2x+1}$



DRV^2 $\quad f''(x) = \dfrac{2 \operatorname{sgn} x \operatorname{sgn}(1-x)(2x-1)}{(2x^2-2x+1)^2}$





2. Polinomio caratteristico: $k^2 - 2\omega k + 1 = 0 \to \begin{cases} k = \omega \pm \sqrt{\omega^2-1} & \text{se } \omega > 1 \\ k = 1 & \text{se } \omega = 1 \\ k = \omega \pm i\sqrt{1-\omega^2} & \text{se } 0 < \omega < 1 \\ k = \pm i & \text{se } \omega = 0 \end{cases}$

Soluzioni omogenea: $\begin{cases} e^{\omega x}\left(c_1 e^{\sqrt{\omega^2-1}\,x} + c_2 e^{-\sqrt{\omega^2-1}\,x}\right) & \text{se } \omega > 1 \\ e^{x}(c_1 + c_2 x) & \text{se } \omega = 1 \\ e^{\omega x}\left(c_1 \cos\sqrt{1-\omega^2}\,x + c_2 \operatorname{sen}\sqrt{1-\omega^2}\,x\right) & \text{se } 0 < \omega < 1 \\ c_1 \cos x + c_2 \operatorname{sen} x & \text{se } \omega = 0 \end{cases}$

Soluzione particolare: passiamo in campo complesso con termine noto e^{ix}; i è radice del polinomio se $\omega = 0$

$\omega \neq 0$ cerchiamo $\bar{z} = A e^{ix} \to A = i/2\omega$
$\bar{z} = \frac{i}{2\omega}(\cos x + i \operatorname{sen} x) \to \bar{y} = -\frac{\operatorname{sen} x}{2\omega}$

$\omega = 0$ cerchiamo $\bar{z} = A x e^{ix} \to A = -\frac{i}{2}$
$\bar{z} = -\frac{i}{2} x (\cos x + i \operatorname{sen} x) \to \bar{y} = \frac{1}{2} x \operatorname{sen} x$

3. $\lg(1+\operatorname{sen} x) = \lg\left(1 + x - \frac{x^3}{6} + o(x^4)\right) = \left(x - \frac{x^3}{6}\right) - \frac{1}{2}\left(x^2 - \frac{x^4}{3}\right) + \frac{1}{3}x^3 - \frac{1}{4}x^4 + o(x^4)$
$= x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + o(x^4)$

$\lg^2(1+\operatorname{sen} x) = x^2 + \frac{1}{4}x^4 - x^3 + \frac{1}{3}x^4 + o(x^4) = x^2 - x^3 + \frac{7}{12}x^4 + o(x^4)$

$N = x^2 - x^3 + \frac{7}{12}x^4 - 1 - x^2 - \frac{1}{2}x^4 + 1 + \frac{1}{2}x^4 + x^3 + o(x^4) = \frac{7}{12}x^4 + o(x^4)$

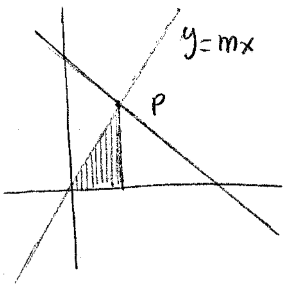
$D = x^2 + o(x^4) - \left(x - \frac{x^3}{6} + o(x^4)\right)^2 = x^2 - x^2 + \frac{x^4}{3} + o(x^4) = \frac{x^4}{3} + o(x^4)$

limite $= \dfrac{7/12}{1/3} = \dfrac{7}{4}$

4. Ponendo $x = \operatorname{sen} t$:

$$\int_0^{\pi/2} \frac{1}{\operatorname{sen} t + 1}\, dt \underset{\operatorname{tg}\frac{t}{2} = z}{=} \int_0^1 \frac{2}{(z+1)^2}\, dt = \left[-\frac{2}{z+1} \right]_0^1 = 1$$

5.



$$P = \left(\frac{1}{m+1}, \frac{m}{m+1} \right), \quad m \geq 0$$

$$V = \frac{1}{3}\pi \frac{m^2}{(m+1)^3} \qquad \text{max per } m = 2.$$

$$S = \pi \frac{m}{m+1} \sqrt{\frac{1+m^2}{(m+1)^2}} = \frac{\pi\, m \sqrt{m^2+1}}{(m+1)^2}$$

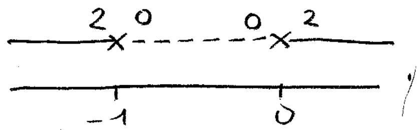
$S' > 0$ dunque S è crescente e non ha max.

Soluzioni [2]

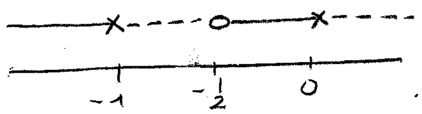
1.
$$\begin{cases} 2x^2+2x+1 > 0 & \rightarrow \forall x \\ |x|/\sqrt{2x^2+2x+1} \leq 1 \longleftrightarrow x^2+2x+1 \geq 0 \rightarrow \forall x \end{cases} \qquad \text{C.E. } \mathbb{R}$$

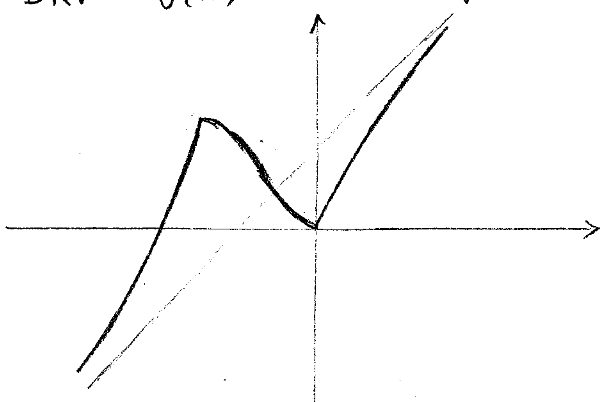
LIM per $x \to \pm\infty$ $f(x) \to \pm\infty$, $y = x + \frac{\pi}{4}$ as. obliquo

DRV $f'(x) = 1 + \dfrac{\operatorname{sgn} x \operatorname{sgn}(x+1)}{2x^2+2x+1}$



DRV^2 $f''(x) = -\operatorname{sgn} x \operatorname{sgn}(x+1) \dfrac{2(2x+1)}{(2x^2+2x+1)^2}$





2. Svolgimento del tutto uguale a quello dato per [1].
Unica differenza: la soluzione particolare $\bar{y}$ è la parte immaginaria di $\bar{z}$ (invece che la parte reale).

3. $$\lg(1+\operatorname{tg} x) = \lg\left(1+x+\frac{x^3}{3}+o(x^4)\right) = \left(x+\frac{x^3}{3}\right) - \frac{1}{2}\left(x^2+\frac{2}{3}x^4\right) + \frac{1}{3}x^3 - \frac{1}{4}x^4 + o(x^4)$$
$$= x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{7}{12}x^4 + o(x^4)$$

$$\lg^2(1+\operatorname{tg} x) = x^2 + \frac{1}{4}x^4 - x^3 + \frac{4}{3}x^4 + o(x^4) = x^2 - x^3 + \frac{19}{12}x^4 + o(x^4)$$

$$N = x^2 - x^3 + \frac{19}{12}x^4 - 1 - x^2 - \frac{1}{2}x^4 + 1 + \frac{1}{2}x^4 + x^3 + o(x^4) = \frac{19}{12}x^4 + o(x^4)$$

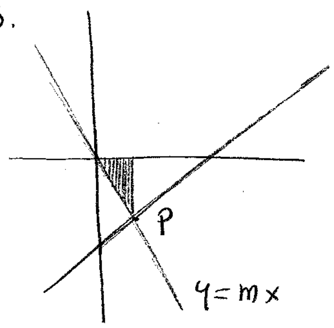
$$D = x^2 + o(x^4) - \left(x + \frac{x^3}{3} + o(x^4)\right)^2 = -\frac{2}{3}x^4 + o(x^4)$$

$$\text{limite} = \frac{19/12}{-2/3} = -\frac{19}{8}.$$

4. Ponendo $x = 2\,\mathrm{sen}\,t$:

$$\int_0^{\pi/2} \frac{1}{2(1+\mathrm{sen}\,t)}\,dt \underset{\mathrm{tg}\frac{t}{2}=z}{=} \int_0^1 \frac{1}{(z+1)^2}\,dz = \left[-\frac{1}{z+1}\right]_0^1 = \frac{1}{2}$$

5.



$$P = \left(\frac{1}{1-m}, \frac{m}{1-m}\right) \qquad m \leq 0.$$

$$V = \frac{1}{3}\pi \frac{m^2}{(1-m)^3} \qquad \text{max per } m = -2$$

$$S = \pi \frac{|m|}{1-m}\sqrt{\frac{1+m^2}{(1-m)^2}} = \frac{-\pi m\sqrt{m^2+1}}{(1-m)^2}$$

$S' < 0$ dunque S è decrescente e non ha max.