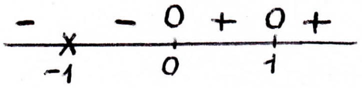
Soluzioni [1]

1. C.E. $x \neq -1$

SGN



LIM per $x \to -1$ $f(x) \to -\infty$ as. verticale

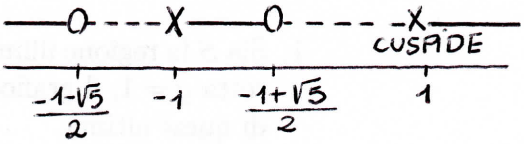
per $x \to \pm\infty$ $f(x) \sim x \to \pm\infty$

per $x \to +\infty$ $f(x) - x = x\,\dfrac{\sqrt{x-1} - \sqrt{x+1}}{\sqrt{x+1}} = \dfrac{-2x}{\sqrt{x+1}\,(\sqrt{x-1} + \sqrt{x+1})}$

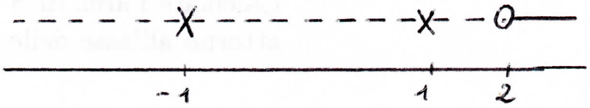
$\sim \dfrac{-2x}{2x} \to -1$; $y = x-1$ as. obliquo

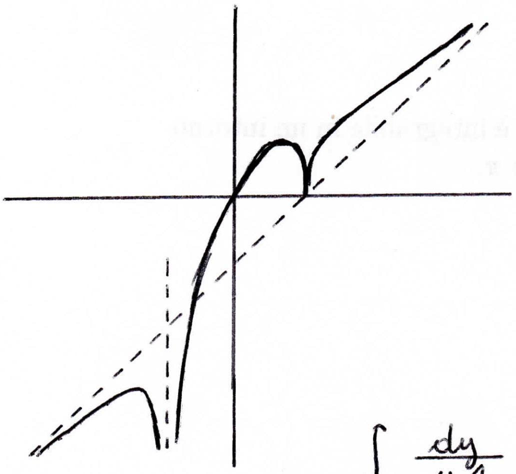
con calcoli analoghi si trova che $y = x-1$ è asintoto anche per $x \to -\infty$.

DRV $f'(x) = \dfrac{x^2+x-1}{x^2-1}\sqrt{\left|\dfrac{x-1}{x+1}\right|}$, $x \neq \pm 1$



DRV^2 $f''(x) = \dfrac{x-2}{(x^2-1)^2}\sqrt{\left|\dfrac{x-1}{x+1}\right|}$





2. $x \in \mathbb{R}$, $y \in \mathbb{R}$

$y = 0$ slz. costante

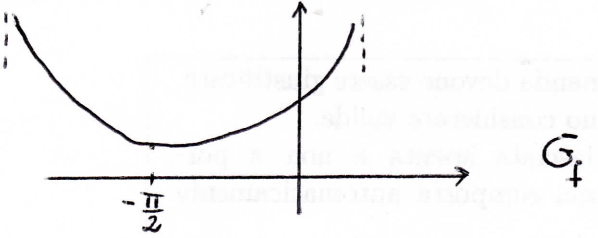
Data la C.I. la studiamo per $y > 0$.
Con le notazioni introdotte a lezione, $B(y) = y^4$, $B'(1)$ esiste: questo assicura l'unicità di soluz. al problema.

$$\int \frac{dy}{y^4} = \int \cos x \, dx \longrightarrow -\frac{1}{3y^3} = \operatorname{sen} x - \frac{c}{3}$$

La C.I. è verificata per $c = 1$.

$$-\frac{1}{3y^3} = \operatorname{sen} x - \frac{1}{3} \longrightarrow y = \frac{1}{\sqrt[3]{1-3\operatorname{sen} x}} ,$$

deve essere $1 - 3\operatorname{sen} x > 0$, cioè $x \in (-\pi - \operatorname{arcsen}\frac{1}{3}, \operatorname{arcsen}\frac{1}{3})$



3.

$$y = -\frac{1}{x}\lg(x^2-2x+2) + 2\int \frac{x-1}{x(x^2-2x+2)}\,dx$$

$$= -\frac{1}{x}\lg(x^2-2x+2) + \int\left(-\frac{1}{x} + \frac{x}{x^2-2x+2}\right)dx$$

$$= -\frac{1}{x}\lg(x^2-2x+2) - \lg|x| + \frac{1}{2}\int \frac{2x-2+2}{x^2-2x+2}\,dx$$

$$= -\frac{1}{x}\lg(x^2-2x+2) - \lg|x| + \frac{1}{2}\lg(x^2-2x+2) + \operatorname{arctg}(x-1) + c$$

4. $f(x) = \sqrt[4]{1-2x^2+x^4} - \cos x = \left[1 + \frac{1}{4}(-2x^2+x^4) - \frac{3}{32}(4x^4)\right] +$

$+ \left[-1 + \frac{1}{2}x^2 - \frac{1}{24}x^4\right] + o(x^4)$

$= -\frac{1}{6}x^4 + o(x^4)$

$x_n \sim -\frac{1}{6}\left(\frac{1}{n}\right)^{4-\alpha}$; la serie converge se $4-\alpha > 1$, cioè $\alpha < 3$.

5. $f(x) = e^{\frac{\lg x}{x}}$; ristretta ai naturali, fornisce la successione

$f'(x) = f(x)\,\frac{1-\lg x}{x^2}$ $\quad \underset{0}{\times}\overset{+}{\text{———}}\underset{e}{|}\overset{-}{\text{———}}$

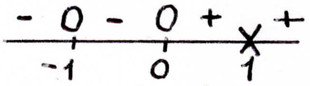
la fz. assume valore massimo per $x = e$; dunque il massimo della successione è il valore più grande tra $x_2 = \sqrt{2}$ e $x_3 = \sqrt[3]{3}$ (essendo $2 < e < 3$) e dunque $\sqrt[3]{3}$.

$\max x_n = \sup x_n = \sqrt[3]{3}$, $\quad \min x_n = \inf x_n = 1$.

Soluzioni [2]

1. C.E. $x \neq 1$

SGN



LIM per $x \to 1$ $f(x) \to +\infty$ as. verticale

per $x \to \pm\infty$ $f(x) \sim x \to \pm\infty$

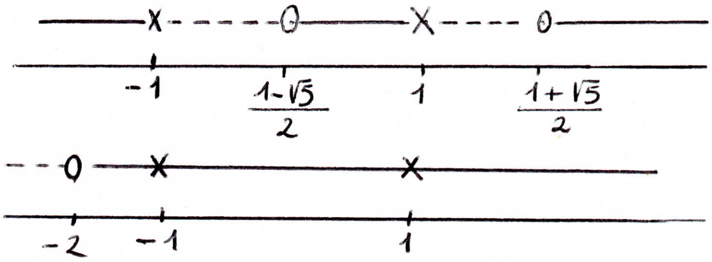
per $x \to +\infty$ $f(x) - x = x \dfrac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x-1}} = \dfrac{2x}{\sqrt{x-1}\,(\sqrt{x+1} + \sqrt{x-1})}$

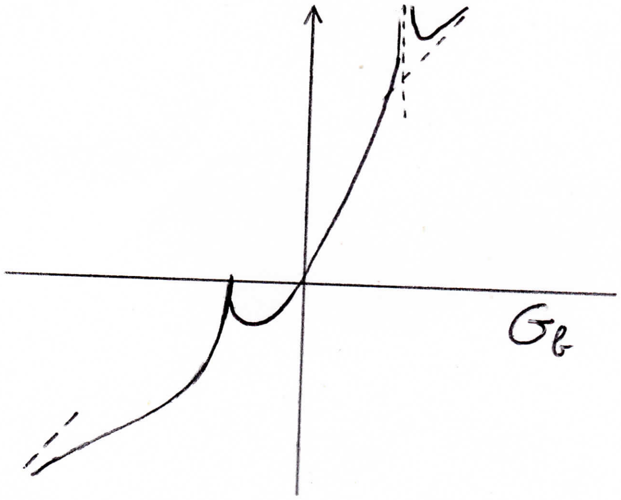
$\sim \dfrac{2x}{2x} \to 1$; $y = x+1$ asintoto obliquo.

Con calcoli analoghi si trova che $y = x+1$ è asintoto anche per $x \to -\infty$.

DRV $f'(x) = \dfrac{x^2 - x - 1}{x^2 - 1} \sqrt{\left|\dfrac{x+1}{x-1}\right|}$, $x \neq \pm 1$

DRV^2 $f''(x) = \dfrac{x+2}{(x^2-1)^2} \sqrt{\left|\dfrac{x+1}{x-1}\right|}$





2. $x \in \mathbb{R}$, $y \in \mathbb{R}$

$y = 0$ sol. costante

Data la C.I. la studiamo per $y > 0$.

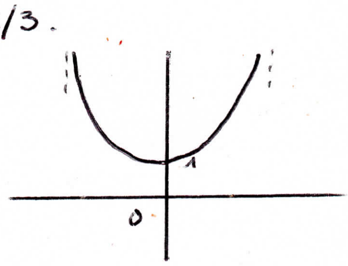
Con le notazioni introdotte a lezione, $\beta(y) = y^4$, $\beta'(1)$ esiste: questo assicura l'unicità di soluzione al problema.

$\int \frac{dy}{y^4} = \int \operatorname{sen} x \, dx \longrightarrow$

$-\dfrac{1}{3y^3} = -\cos x - c$; la C.I. è verificata per $c = -2/3$.

$-\dfrac{1}{3y^3} = -\cos x + \dfrac{2}{3} \rightarrow y = \dfrac{1}{\sqrt[3]{3\cos x - 2}}$;

dove essere $3 \cos x - 2 > 0$, cioè $|x| < \arccos \frac{2}{3}$



3. $y = -\dfrac{1}{x} \lg(x^2+2x+2) + 2 \displaystyle\int \dfrac{x+1}{x(x^2+2x+2)} dx =$

$= -\dfrac{1}{x} \lg(x^2+2x+2) + \displaystyle\int \left(\dfrac{1}{x} - \dfrac{x}{x^2+2x+2} \right) dx =$

$= -\dfrac{1}{x} + \lg|x| - \dfrac{1}{2} \displaystyle\int \dfrac{2x+2-2}{x^2+2x+2} dx =$

$= -\dfrac{1}{x} + \lg|x| - \dfrac{1}{2} \lg(x^2+2x+2) + \operatorname{arctg}(x+1) + c$

4. Vedi [1]

5. $f(x) = e^{\frac{\lg x}{x^2}}$, $x>0$.

$f'(x) = f(x)\,\frac{1-2\lg x}{x^3}$ $\quad \underset{0}{\times}\overset{+}{\overline{\qquad\quad}}\underset{\sqrt{e}}{|}\overset{-}{\overline{\qquad\quad}}$

La fz. assume massimo per $x=\sqrt{e}$; dunque la successione ha come massimo il più grande tra $x_1 = 1$ e $x_2 = \sqrt[4]{2}$ (essendo $1<\sqrt{e}<2$), e dunque $\sqrt[4]{2}$.

max x_n = sup x_n = $\sqrt[4]{2}$, min x_n = inf x_n = 1.