Soluzioni [1]

1. Funzione dispari $(-f(-x) = f(x))$

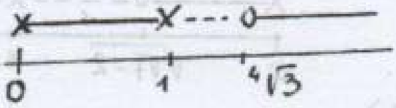
C.E. $x \neq 0$, $x \neq \pm 1$. Tenendo conto della simmetria, possiamo limitarci a studiarla per $x \in (0,1) \cup (1,+\infty)$.

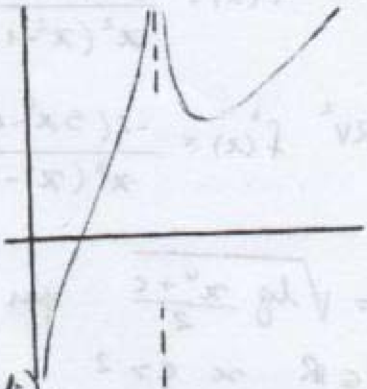
LIM. per $x \to 0^+$ $f(x) \to -\infty$ as. verticale

per $x \to 1$ $f(x) \to +\infty$ as. verticale

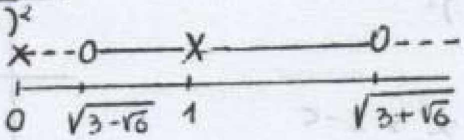
per $x \to +\infty$ $f(x) = x + o(1) \Rightarrow f(x) \to +\infty$ con asintoto $y = x$.

DRV $f'(x) = \dfrac{x^4 - 3}{x^2(x^2-1)}$





DRV^2 $f''(x) = \dfrac{-2(x^4 - 6x^2 + 3)}{x^3(x^2-1)^2}$



2. eq. diff a variabili separate

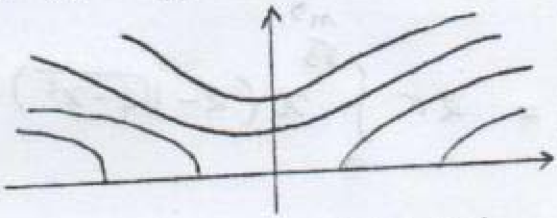
C.E. $x \in \mathbb{R}$, $y > 0$ (questa condizione è data dal lg).

Non ci sono slz. costanti.

$\int y e^{y^2} dy = \int x^5 dx \to \frac{1}{2} e^{y^2} = \dfrac{x^6 + c}{6} \to y = \sqrt{\lg \dfrac{x^6+c}{3}}$

definite per $\dfrac{x^6+c}{3} > 1$, cioè $x^6 > 3 - c$.

Dunque, per $x \in \mathbb{R}$ se $c > 3$, per $|x| > \sqrt[6]{3-c}$ se $c \leq 3$.



3. $\operatorname{arctg} x = x - \dfrac{x^3}{3} + o(x^4)$

$\operatorname{sen} x = x - \dfrac{x^3}{6} + o(x^4)$

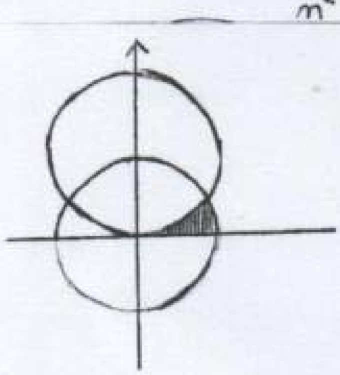
$\operatorname{tg} x = x + \dfrac{x^3}{3} + o(x^4)$

$\lg(n^2+1) - \lg n^2 = \lg\left(1 + \frac{1}{n^2}\right) \sim \frac{1}{n^2}$

$x_n \sim \dfrac{-\frac{1}{6n^3} \cdot \left(\frac{1}{3n^3}\right)^{\alpha}}{\frac{1}{n^2}} \sim \dfrac{K}{n^{1+3\alpha}}$

la serie converge se $1 + 3\alpha > 1$, cioè se $\alpha > 0$.

4.



Le due circonferenze si intersecano (nel primo quadrante) nel punto $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$A = 2\pi \int_0^{\sqrt{3}/2} x\left(1 - \sqrt{1-x^2}\right) dx + 2\pi \int_{\sqrt{3}/2}^{1} x\sqrt{1-x^2}\, dx =$

$= \pi/4$

$\int x\sqrt{1-x^2}\, dx \underset{\substack{1-x^2 = t \\ -2x\,dx = dt}}{=} -\frac{1}{2}\int \sqrt{t}\, dt = -\frac{1}{3} t^{3/2} + c = -\frac{1}{3}(1-x^2)^{3/2} + c.$

Soluzioni [2]

Riportiamo i calcoli principali, lo svolgimento essendo del tutto analogo a quello visto in [1]

1.

fz. dispari

CE. $x \neq 0$, $x \neq \pm 1$

LIM per $x \to 0^+$ $f(x) \to -\infty$

$x \to 1$ $f(x) \to -\infty$

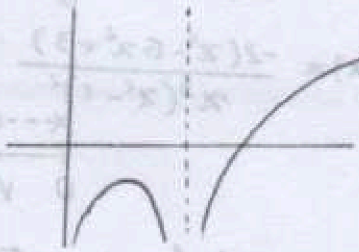
$x \to +\infty$ $f(x) = x + o(1)$

DRV $$f'(x) = \frac{x^4 + 4x^2 - 3}{x^2(x^2-1)}$$

x———0- - - -x———

$0 \quad \sqrt{\sqrt{7}-2} \quad 1$

DRV² $$f''(x) = \frac{-2(5x^4 - 6x^2 + 3)}{x^3(x^2-1)^2} \to \text{sempre} < 0$$



2. $$y = \sqrt{\lg \frac{x^4 + c}{2}} \quad \text{per } x^4 > 2 - c$$

$x \in \mathbb{R}$ se $c > 2$

$|x| > \sqrt[4]{2-c}$ se $c \leq 2$ Grafici analoghi a quelli in [1]

3. $\arcsin x = x + \frac{x^3}{6} + o(x^4)$

$$x_n \sim \frac{-\frac{1}{6n^3} \cdot \left(-\frac{1}{6n^3}\right)^\alpha}{\frac{1}{n^3}} \sim \frac{c}{n^{3\alpha}}$$ converge se $3\alpha > 1$, cioè $\alpha > \frac{1}{3}$

4. $$A = 2\pi \int_0^{\sqrt{3}} x\left(2 - \sqrt{4-x^2}\right) dx + 2\pi \int_{\sqrt{3}}^{2} x\sqrt{4-x^2}\, dx = \dots = 2\pi$$