Soluzioni

1.

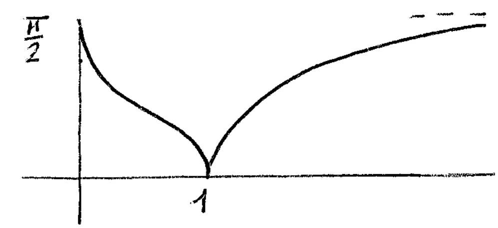
C.E. $\quad \left|\dfrac{x-1}{x+1}\right| \leq 1 \iff \begin{cases} x \neq -1 \\ x^2-2x+1 \leq x^2+2x+1 \end{cases} \iff x \geq 0$

SGN $\quad$ positiva ; nulla per $x = 1$

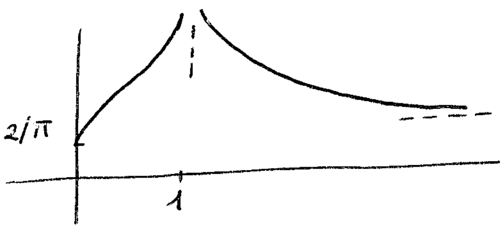
LIM $\quad$ per $x \to +\infty \quad f(x) \to \frac{\pi}{2}$; $f(0) = \frac{\pi}{2}$

DRV $\quad f'(x) = \dfrac{1}{\sqrt{1-\left|\frac{x-1}{x+1}\right|}} \sqrt{\left|\dfrac{x+1}{x-1}\right|} \operatorname{sgn}\left(\dfrac{x-1}{x+1}\right) \dfrac{1}{(x+1)^2}$ $\quad$ (segno: $-$ tra 0 e 1, $+$ per $x>1$)

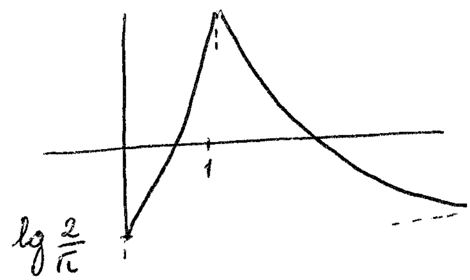
$x = 1$ cuspide $\qquad x = 0$ punto a tg verticale



G_f



$G_{\frac{1}{f}}$



2. Con le notazioni standard : $a(x) = -2x$, $A(x) = -x^2$

$$(y e^{-x^2})' = 2x^3 e^{-x^2}$$

$$\int 2x^3 e^{-x^2} dx \underset{\substack{-x^2 = t \\ -2x\,dx = dt}}{=} \int t e^t dt \underset{\text{per parti}}{=} e^t(t-1) + c = -e^{-x^2}(x^2+1) + c$$

$$y e^{-x^2} = -(x^2+1)e^{-x^2} + c \Rightarrow y = c e^{x^2} - x^2 - 1.$$

$y(0) = 1$ per $c = 2$.

3. $a_1 = -4$, $a_2 = 13/3$

$a_n > 0 \quad \forall n \geq 2$

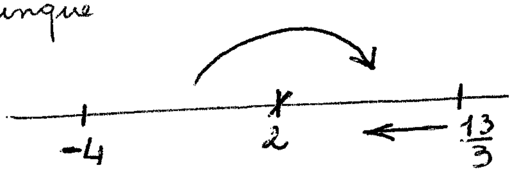
$$a_{n+1} \geq a_n \iff \frac{7a_n+2}{2a_n+4} \geq a_n \iff 2a_n^2 - 3a_n - 2 \leq 0$$

$$\iff 0 < a_n \leq 2.$$

Poichè $a_2 > 2$, proviamo per induzione che è $a_n > 2$.

$$\frac{7a_n+2}{2a_n+4} \geq 2 \iff 7a_n + 2 \geq 4a_n + 8 \iff 3a_n \geq 6$$

Dunque



min = inf = -4

max = sup = $13/3$

lim = 2

4. $e^{i5\pi/2} = i$

$\cos\frac{\pi}{3} + i \operatorname{sen}\frac{\pi}{3} = \frac{1+i\sqrt{3}}{2}$

$(i+1)^8 = (\sqrt{2}\, e^{i\pi/4})^8 = 16$

$2\sqrt{3} - 1 - 2i - \sqrt{3}i = 2\sqrt{3} + i^2 - 2i - \sqrt{3}i = \sqrt{3}(2-i) + i(i-2) = (i-\sqrt{3})(i-2)$

$$\frac{32(i-2)(A+iy)\, i\, (1+i\sqrt{3})\frac{1}{2}}{16(i-2)(i-\sqrt{3})} = A + iy.$$

$$\int \frac{4x^3+2x}{x^4+x^2+3} \lg^3(x^4+x^2+3)\, dx = \int t^3 dt$$

$\lg(x^4+x^2+3) = t$

$\frac{4x^3+2x}{x^4+x^2+3}\, dx = dt$

$$= \frac{1}{4}t^4 + c = \frac{1}{4}\lg^4(x^4+x^2+3) + c.$$