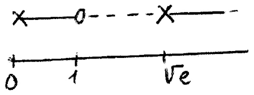
Soluzioni [1]

1. $f(x)$ dispari $\Rightarrow$ si studia per $x \geq 0$: $f(x) = \dfrac{x \lg x}{2 \lg x - 1}$.

C.E. $\quad x > 0, \ x \neq \sqrt{e}$

SGN



LIM $\quad$ per $x \to 0$ $\quad f(x) \sim \dfrac{x \lg x}{2 \lg x} \to 0$ DISC. ELIM.

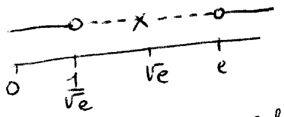
per $x \to \sqrt{e}^{\pm}$ $\quad f(x) \to \pm\infty$ AS. VERTICALE

per $x \to +\infty$ $\quad f(x) \sim \dfrac{x \cancel{\lg x}}{2 \cancel{\lg x}} \to +\infty$

$f(x) - \dfrac{x}{2} = \dfrac{x}{2(2\lg x - 1)} \to +\infty$

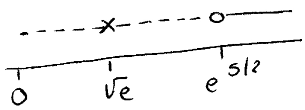
non c'è asint.

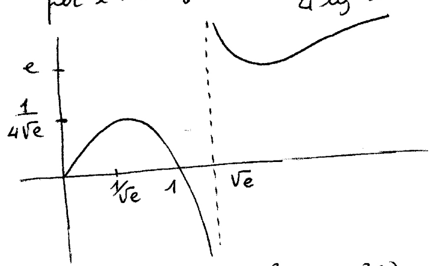
DRV. $\quad f'(x) = \dfrac{2 \lg^2 x - \lg x - 1}{(2 \lg x - 1)^2}$



per $x \to 0$ $\quad f'(x) \sim \dfrac{2 \lg^2 x}{4 \lg^2 x} \to \dfrac{1}{2}$; $\quad f'(0) = \dfrac{1}{2}$

DRV^2 $\quad f''(x) = \dfrac{5 - 2 \lg x}{x (2 \lg x - 1)^3}$





2. $\lg\left(1 + x - \frac{x^3}{6} + o(x^3)\right) - \lg\left(1 - x + \frac{x^3}{6} + o(x^3)\right) - 2\left(x - \frac{x^3}{6} + o(x^3)\right) - \left(x - \frac{x^3}{6} + o(x^3)\right)^3$

$\left(x - \frac{x^3}{6}\right) - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \left(-x + \frac{1}{6} x^3\right) + \frac{1}{2} x^2 - \frac{1}{3}(-x^3) - 2x + \frac{x^3}{3} - x^3 + o(x^3)$

$- \frac{1}{3} x^3 + o(x^3)$.

3. $e^{\frac{\lg(\operatorname{tg} x / x)}{x}} \sim e^{\frac{\operatorname{tg} x / x - 1}{x}} = e^{\frac{\operatorname{tg} x - x}{x^2}} \sim e^{\frac{x^3/3}{x^2}} = e^{x/3} \to 1$.

$\dfrac{\operatorname{sen} x}{\lg(1+x)} \sim \dfrac{x}{x} \to 1$

Prolungamento continuo : si pone $f(0) = 1$.

$\dfrac{e^{\frac{\lg(\operatorname{tg} x / x)}{x}} - 1}{x} \sim \dfrac{e^{x/3} - 1}{x} \sim \dfrac{x/3}{x} \to \dfrac{1}{3}$

$\dfrac{\frac{\operatorname{sen} x}{\lg(1+x)} - 1}{x} = \dfrac{\operatorname{sen} x - \lg(1+x)}{x \lg(1+x)} \sim \dfrac{x^2/2}{x^2} \to \dfrac{1}{2}$

$f'(0)$ non esiste (punto angoloso).

1. $f(x)$ dispari $\Rightarrow$ si studia per $x \geq 0$: $f(x) = \dfrac{2x\lg x}{1-\lg x}$

C.E. $\quad x > 0,\ x \neq e$

SGN

```
x-----o------x-----
|     |      |
0     1      e
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LIM per $x \to 0$ $\quad f(x) \sim \dfrac{2x\lg x}{-\lg x} \to 0$ D.E.

per $x \to e^{\pm}$ $\quad f(x) \to \mp\infty$ AS. VERTIC.

per $x \to +\infty$ $\quad f(x) \sim \dfrac{2x\lg x}{-\lg x} \sim -2x \to -\infty$

$f(x) + 2x = \dfrac{2x}{1-\lg x} \to -\infty$ non c'è asintoto

DRV $\quad f'(x) = \dfrac{2(1+\lg x - \lg^2 x)}{(1-\lg x)^2}$

```
----o--x--o-----
|   |  |  |
0   α  e  β
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$\alpha = e^{\frac{1-\sqrt5}{2}}$

$\beta = e^{\frac{1+\sqrt5}{2}}$

per $x \to 0$ $\quad f'(x) \sim \dfrac{-2\lg^2 x}{\lg^2 x} \to -2$

$\Downarrow$

$f'(0) = -2$

DRV² $\quad f''(x) = \dfrac{2(3-\lg x)}{x(1-\lg x)^3}$

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------x-----o----
|     |     |
0     e     e^3
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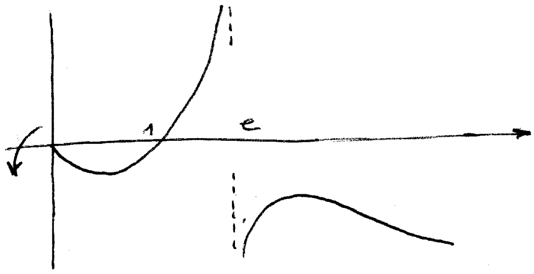

2. La funzione è l'opposta di quella studiata in [1]. I calcoli si deducono senza difficoltà da quelli riportati precedentemente.

$f(x) \sim \frac13 x^3$

3. $e^{\frac{\lg(\operatorname{sen}x/x)}{x}} \sim e^{\frac{\frac{\operatorname{sen}x}{x}-1}{x}} = e^{\frac{\operatorname{sen}x - x}{x^2}} \sim e^{\frac{-x^3/6}{x^2}} = e^{-x/6} \to 1$

$\dfrac{\operatorname{tg}x}{\sqrt{1+2x}-1} \sim \dfrac{x}{x} \to 1 \qquad f(0) = 1.$

$\dfrac{e^{\frac{\lg(\operatorname{sen}x/x)}{x}} - 1}{x} \sim \dfrac{\frac{\lg(\operatorname{sen}x/x)}{x}}{x} \sim \dfrac{\operatorname{sen}x - x}{x^3} \sim \dfrac{-x^3/6}{x^3} \to -\dfrac16$

$\dfrac{\frac{\operatorname{tg}x}{\sqrt{1+2x}-1} - 1}{x} = \dfrac{\operatorname{tg}x - \sqrt{1+2x} + 1}{x(\sqrt{1+2x}-1)} = \dfrac{(x + o(x^2)) - (1 + x - \frac12 x^2 + o(x^2)) + 1}{x(x + o(x))}$

$\simeq \dfrac{\frac12 x^2}{x^2} \to \dfrac12 \qquad f'(0)$ non esiste.