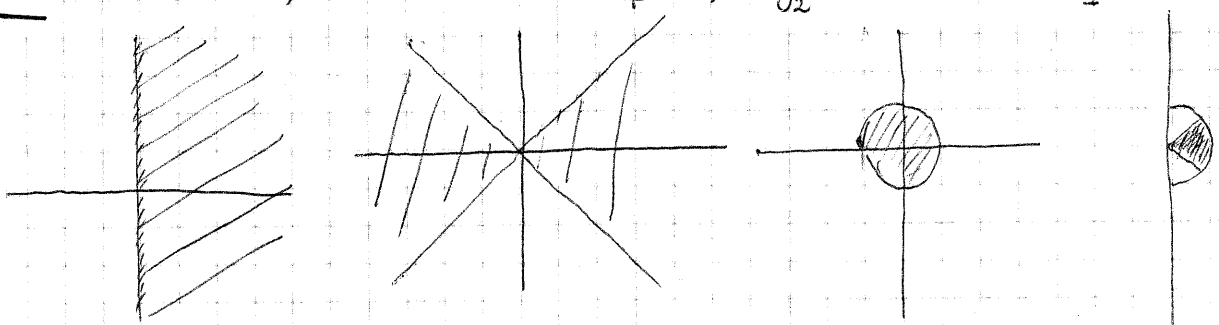
1.A $\log_{\sqrt3}2+\log_{\sqrt3}(2-\frac12)+\log_{\sqrt3}(2-\frac13)+\log_{\sqrt3}(2-\frac15)=\log_{\sqrt3}2+\log_{\sqrt3}\frac32+\log_{\sqrt3}\frac53+\log_{\sqrt3}\frac95$

$=\log_{\sqrt3}\left(2\cdot\frac32\cdot\frac53\cdot\frac95\right)=\log_{\sqrt3}9=\log_{\sqrt3}3^2=\log_{\sqrt3}(\sqrt3)^4=4\log_{\sqrt3}\sqrt3=4$

2.A $\sqrt{x}\Rightarrow x\geq 0$, $\ln(|x|-|y|)\Rightarrow |x|\gneqq|y|$, $\log_2(1-x^2-y^2)\Rightarrow 1\gneqq x^2+y^2$



3A Sia X = contenuto iniziale = rifornimento settimanale $= 3400\,l$
sia α = residuo percentuale acqua a fine settimana $=\frac{15}{100}=\frac{3}{20}$

A_{n+1} = acqua presente all'inizio della settimana $(n+1)^a$ $= X+\alpha A_n$

$= X+\alpha(X+\alpha A_{n-1}) = X+\alpha X+\alpha^2 A_{n-1} = X+\alpha X+\alpha^2 X+\alpha^3 A_{n-2} =$

$=\dots = X+\alpha X+\alpha^2X+\alpha^3X+\dots+\alpha^nX$

$= X(1+\alpha+\dots+\alpha^n) = X\,\frac{1-\alpha^{n+1}}{1-\alpha}\leq\frac{X}{1-\alpha}=\frac{3400}{\frac{85}{100}}\,l=\frac{3400}{\frac{17}{20}}\,l=$

$\leq \boxed{4000\,l}$

4.A $\frac{2^n+1}{2-4^n}=\frac{2^n}{4^n}\cdot\frac{1+\frac{1}{2^n}}{\frac{2}{4^n}-1}=\left(\frac24\right)^n\cdot\frac{1+\frac{1}{2^n}}{\frac{2}{4^n}-1}=\left(\frac12\right)^n\cdot\frac{1+\left(\frac12\right)^n}{2\left(\frac14\right)^n-1}$ $\boxed{\longrightarrow 0 \quad n\to+\infty}$

5.A
$A\triangle B=(A\cup B)\setminus(A\cap B)=(A\setminus B)\cup(B\setminus A)$

$A=\left\{x:\ \frac{2-4^{x^2+\frac{x}{2}}}{3^x+2}\geq 0\right\}=$ [poiché $3^x+2\gneqq 0$ per ogni x] $=\left\{x:\ 2-4^{x^2+\frac{x}{2}}\geq 0\right\}=$

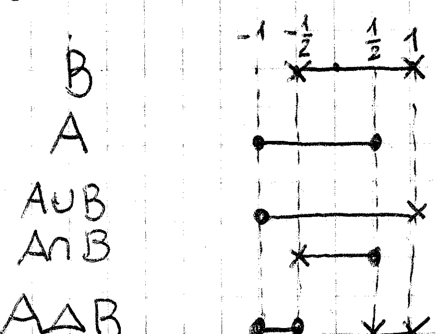
$=\left\{x:\ 4^{\frac12}-4^{x^2+\frac{x}{2}}\geq 0\right\}=\left\{x:\ 1-4^{x^2+\frac{x}{2}-\frac12}\geq 0\right\}=\left\{x:\ x^2+\frac{x}{2}-\frac12\leq 0\right\}$

$B=\left\{x:\ \frac{\log_3(2x+1)-1}{x^2+2x+2}<0\right\}=$ [poiché $x^2+2x+2=(x+1)^2+1$]

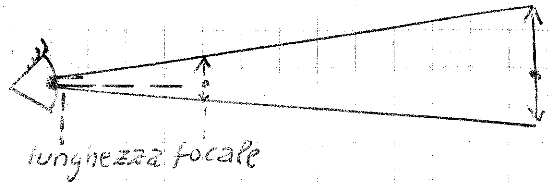
$=\left\{x:\ \log_3(2x+1)-1<0\right\}=\left\{x:\ \log_3(2x+1)<1 \text{ e } 2x+1>0\right\}=$

$=\left\{x:\ 2x+1<3 \text{ e } x>-\frac12\right\}=\left\{x:\ x<1 \text{ e } x>-\frac12\right\}=\left]-\frac12;1\right[$

$A=\left\{x:\ x^2+x/2-1/2\leq 0\right\}=\left[\ \frac{-1/2\pm\sqrt{1/4+2}}{2}=\left\{-\frac14\pm\frac34\right\}=\left\{-1,\frac12\right\}\right]=\left[-1,\frac12\right]$



$\boxed{A\triangle B=\left[-1;-\frac12\right]\cup\left(\frac12;1\right)}$

6.A 

Il fattore di ingrandimento è dato da $\dfrac{\text{distanza}}{\text{lunghezza focale}}$, (linearmente)

quindi per le aree il fattore è $\left(\dfrac{\text{dist.}}{\text{lung. foc.}}\right)^2$.

Nel caso conviene esprimere l'errore relativo con le misure massime e minime:

$$e_{rel} = \frac{\text{Area massima} - \text{Area minima}}{\text{Area massima} + \text{Area minima}}, \quad \text{errore percentuale} = e_{rel}\cdot 100$$

$$\text{Area massima} = \left(\frac{\text{dis. max}}{\text{lun. foc. min}}\right)^2 (\text{Primo lato massimo})(\text{secondo lato massimo})$$

$$\text{Area minima} = \left(\frac{\text{dis. min.}}{\text{lun. foc. max}}\right)^2 (\text{primo lato min})(\text{secondo lato minimo})$$

Esprimendo in cm i dati: $\text{dist} = (4{,}5 \pm 3)\cdot 10^3$ cm, $\text{lun. foc.} = 26 \pm 2$ cm

primo $\ell = 2{,}8 \div 4{,}2$ secondo $\ell = 3 \div 4{,}5$

$$\left(\frac{d.\,max}{\ell.f.\,min}\right)^2 = \left(\frac{48}{24}\cdot 10^3\right)^2 = 4\cdot 10^6 \qquad \left(\frac{d.\,min}{\ell.f.\,max}\right)^2 = \left(\frac{42}{28}\cdot 10^3\right)^2 = \frac{9}{4}\,10^6$$

$$e_{rel} = \frac{4\cdot 10^6\,\frac{42}{10}\cdot\frac{45}{10} - \frac{9}{4}\,10^6\,\frac{28}{10}\cdot 3}{4\,10^6\,\frac{42}{10}\cdot\frac{45}{10} + \frac{9}{4}\,10^6\,\frac{28}{10}\cdot 3} = \frac{16\cdot 42\cdot 45 - 9\cdot 28\cdot 30}{16\cdot 42\cdot 45 + 9\cdot 28\cdot 30}$$

$$= \frac{16\cdot 42\cdot 3 - 9\cdot 28\cdot 2}{16\cdot 42\cdot 3 + 9\cdot 28\cdot 2} = \frac{16\cdot 42 - 3\cdot 28\cdot 2}{16\cdot 42 + 3\cdot 28\cdot 2} = \frac{2\cdot 42 - 3\cdot 7}{2\cdot 42 + 3\cdot 7} = \frac{4-1}{4+1} = \frac{3}{5}$$

$\boxed{\text{Errore percentuale} = 60\%}$

7.A Sia T la quantità di individui in ciascuna colonia.

Dopo 4 ore la prima colonia ha $T + \frac{T}{10} + \frac{1}{10}\left(T + \frac{T}{10}\right) = T\left(1+\frac{1}{10}\right)^2$ individui

la seconda colonia ha $T + \frac{T}{10}$ individui

in totale $T\left(1 + \frac{1}{10} + \left(1+\frac{1}{10}\right)^2\right) = T\left(2 + \frac{3}{10} + \frac{1}{100}\right) < 2T + 2T\cdot\frac{25}{100}$

Dopo altre 4 ore la prima colonia arriva a $T\left(1+\frac{1}{10}\right)^4$ individui

la seconda colonia a $T\left(1+\frac{1}{10}\right)^2$, per un totale di

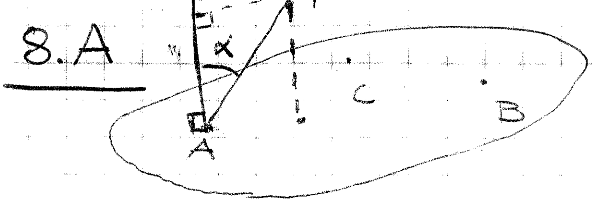
$$T\left(1+\frac{1}{10}\right)^2\left(\left(1+\frac{1}{10}\right)^2+1\right) = T\left(\frac{11}{10}\right)^2\cdot\left(\left(\frac{11}{10}\right)^2+1\right) = T\,\frac{121}{100}\cdot\frac{221}{100}$$

da confrontare con $T\left(2+\frac{50}{100}\right) = T\,\frac{250}{100}$

Ma $121\cdot 221 = 26.741 > 25000$

Quindi tra $\boxed{\text{le 4 e le 8 ore}}$ la popolazione aumenta di 1/4

8.A



La distanza cercata è $\|P-A\| \cdot |\cos\alpha| = d$

Ma $\|P-A\|\,|\cos\alpha| = \left|(P-A) \cdot \dfrac{(B-A)\times(C-A)}{\|(B-A)\times(C-A)\|}\right|$

$P-A = (0,-3,0)$, $B-A = (0,0,1)$, $C-A = (-2,1,0)$

$(B-A)\times(C-A) = (-1,-2,0)$ di norma $\sqrt{1+4} = \sqrt{5}$

Quindi $\boxed{d = \dfrac{6}{\sqrt{5}}}$.

9.A Se R_P è la rotazione attorno a P e R_0 attorno a $(0,0)$

$$R_P Q = R_0(Q-P) + P$$

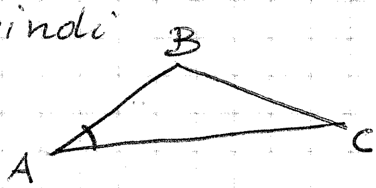
$R_0 = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ $\qquad Q-P = (2,-1)-(1,1) = (2-1,-1-1) = (1,-2)$

$$\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}+\sqrt{3} \\ \frac{\sqrt{3}}{2} - 2\cdot\frac{1}{2} \end{pmatrix} = \left(\frac{1}{2}+\sqrt{3}, \frac{\sqrt{3}}{2}-1\right)$$

Le coordinate cercate sono $\boxed{\left(\frac{3}{2}+\sqrt{3}, \frac{\sqrt{3}}{2}\right)}$.

10.A Trattandosi dell'angolo di un triangolo il suo seno sarà non negativo, quindi

$\sin\hat{A} = \sqrt{1-\cos^2\hat{A}}$



$$|\cos\hat{A}| = \frac{|(B-A)\cdot(C-A)|}{\|B-A\|\,\|C-A\|} = \frac{|(-1,4,1)\cdot(-4,2,0)|}{\sqrt{1+16+1}\,\sqrt{16+4}} =$$

$$= \frac{4+8}{\sqrt{18}\sqrt{20}} = \frac{12}{3\sqrt{2}\,2\sqrt{5}} = \frac{2}{\sqrt{10}}$$

$$\sin\hat{A} = \sqrt{1-\frac{4}{10}} = \sqrt{\frac{6}{10}} = \boxed{\sqrt{\frac{3}{5}}}.$$